Algebra – Lösungen zu Übungsserie 10

1. (a) $\sqrt{32} = \sqrt{2^5} = 2^2\sqrt{2} = \underline{\underline{4\sqrt{2}}}$

   (b) $\sqrt{169} = \underline{\underline{13}}$

   (c) $\sqrt{243} = \sqrt{3^5} = 3^2\sqrt{3} = \underline{\underline{9\sqrt{3}}}$

   (d) $\sqrt{288} = \sqrt{2^5 \cdot 3^2} = 2^2 \cdot 3\sqrt{2} = \underline{\underline{12\sqrt{2}}}$ oder $\sqrt{288} = \sqrt{2 \cdot 144} = \underline{\underline{12\sqrt{2}}}$

   (e) $\sqrt{512} = \sqrt{2^9} = 2^4\sqrt{2} = \underline{\underline{16\sqrt{2}}}$

   (f) $\sqrt{1024} = \sqrt{2^{10}} = 2^5 = \underline{\underline{32}}$

2. (a) $\sqrt{\frac{99}{25}} = \frac{\sqrt{9 \cdot 11}}{\sqrt{25}} = \underline{\underline{\frac{3\sqrt{11}}{5}}}$

   (b) $\sqrt{4\,\frac{4}{9}} = \sqrt{\frac{40}{9}} = \frac{\sqrt{4 \cdot 10}}{3} = \underline{\underline{\frac{2\sqrt{10}}{3}}}$

   (c) $\sqrt{\frac{1}{50}} = \frac{1}{\sqrt{25 \cdot 2}} = \frac{1}{5\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{5 \cdot 2} = \underline{\underline{\frac{\sqrt{2}}{10}}}$ kürzer: $\sqrt{\frac{1}{50}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{100}} = \underline{\underline{\frac{\sqrt{2}}{10}}}$

   (d) $\sqrt{\frac{64}{45}} = \frac{\sqrt{64}}{\sqrt{9 \cdot 5}} = \frac{8}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{8\sqrt{5}}{3 \cdot 5} = \underline{\underline{\frac{8\sqrt{5}}{15}}}$

   (e) $\sqrt{3\,\frac{3}{8}} = \sqrt{\frac{27}{8}} = \frac{3\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \underline{\underline{\frac{3\sqrt{6}}{4}}}$

   (f) $\sqrt{\frac{99}{35}} = \frac{\sqrt{9 \cdot 11}}{\sqrt{5 \cdot 7}} \cdot \frac{\sqrt{5 \cdot 7}}{\sqrt{5 \cdot 7}} = \frac{3\sqrt{11 \cdot 5 \cdot 7}}{5 \cdot 7} = \underline{\underline{\frac{3\sqrt{385}}{35}}}$

3. (a) $\frac{3}{4}\sqrt{7} + 1.5\sqrt{7} = \left(\frac{3}{4} + \frac{3}{2}\right)\sqrt{7} = \underline{\underline{\frac{9}{4}\sqrt{7}}}$

   (b) $\sqrt{20} + \sqrt{5} = \sqrt{4 \cdot 5} + \sqrt{5} = 2\sqrt{5} + \sqrt{5} = \underline{\underline{3\sqrt{5}}}$

   (c) $2 + \sqrt{2} = \underline{\underline{2 + \sqrt{2}}}$ keine weitere Vereinfachung möglich

   (d) $\sqrt{2} + \sqrt{3} = \underline{\underline{\sqrt{2} + \sqrt{3}}}$ keine weitere Vereinfachung möglich

   (e) $\sqrt{3} : \sqrt{8} = \frac{\sqrt{3}}{\sqrt{8}} = \frac{\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \underline{\underline{\frac{\sqrt{6}}{4}}}$

4. (a) $3+\sqrt{3}+9+\sqrt{9}+27+\sqrt{27}=39+\sqrt{3}+3+3\sqrt{3}=\underline{\underline{42+4\sqrt{3}}}$

(b) $\sqrt{\frac{1}{3}}+\sqrt{\frac{3}{4}}-\sqrt{\frac{1}{12}}=\frac{\sqrt{3}}{3}+\frac{\sqrt{3}}{2}-\frac{1}{2\sqrt{3}}=\frac{\sqrt{3}}{3}+\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{6}=\left(\frac{1}{3}+\frac{1}{2}-\frac{1}{6}\right)\sqrt{3}=\underline{\underline{\frac{2\sqrt{3}}{3}}}$

(c) $2\sqrt{0.5}+3\sqrt{\frac{1}{3}}-\frac{1}{2}\sqrt{18}+0.25\sqrt{27}=\frac{2}{\sqrt{2}}+\frac{3}{\sqrt{3}}-\frac{\sqrt{9\cdot 2}}{2}+\frac{\sqrt{9\cdot 3}}{4}$

$=\sqrt{2}+\sqrt{3}-\frac{3\sqrt{2}}{2}+\frac{3\sqrt{3}}{4}=\underline{\underline{-\frac{\sqrt{2}}{2}+\frac{7\sqrt{3}}{4}}}$

(d) $\sqrt{\frac{1}{2}+\frac{1}{4}}+\sqrt{\frac{1}{2}-\frac{1}{4}}+\sqrt{\frac{1}{2}\cdot\frac{1}{4}}+\sqrt{\frac{1}{2}:\frac{1}{4}}=\sqrt{\frac{3}{4}}+\sqrt{\frac{1}{4}}+\sqrt{\frac{1}{8}}+\sqrt{2}$

$=\frac{\sqrt{3}}{2}+\frac{1}{2}+\frac{1}{2\sqrt{2}}+\sqrt{2}=\frac{\sqrt{3}}{2}+\frac{1}{2}+\frac{\sqrt{2}}{4}+\sqrt{2}=\underline{\underline{\frac{1}{2}+\frac{5\sqrt{2}}{4}+\frac{\sqrt{3}}{2}}}$

5. (a) $\frac{1}{4}\sqrt{3}\cdot 3\sqrt{6}=\frac{3}{4}\sqrt{18}=\underline{\underline{\frac{9\sqrt{2}}{4}}}$

(b) $\sqrt{6}\left(\sqrt{2}-\sqrt{6}-\sqrt{12}\right)=\sqrt{12}-6-\sqrt{36\cdot 2}=\underline{\underline{-6-6\sqrt{2}+2\sqrt{3}}}$

(c) $\left(\sqrt{10}+\sqrt{5}\right)^2=10+2\sqrt{50}+5=\underline{\underline{15+10\sqrt{2}}}$

(d) $\frac{3}{2\sqrt{6}}=\frac{3\sqrt{6}}{2\cdot 6}=\underline{\underline{\frac{\sqrt{6}}{4}}}$

(e) Klammern paarweise ausmultiplizieren, binomische Formeln helfen!

$\left(\sqrt{3}+\sqrt{5}+\sqrt{7}\right)\left(\sqrt{3}+\sqrt{5}-\sqrt{7}\right)\left(\sqrt{3}-\sqrt{5}+\sqrt{7}\right)\left(-\sqrt{3}+\sqrt{5}+\sqrt{7}\right)$

$=\Big(\left(\sqrt{3}+\sqrt{5}\right)+\sqrt{7}\Big)\Big(\left(\sqrt{3}+\sqrt{5}\right)-\sqrt{7}\Big)\Big(\sqrt{7}+\left(\sqrt{3}-\sqrt{5}\right)\Big)\Big(\sqrt{7}-\left(\sqrt{3}-\sqrt{5}\right)\Big)$

$=\Big(\left(\sqrt{3}+\sqrt{5}\right)^2-\left(\sqrt{7}\right)^2\Big)\Big(\left(\sqrt{7}\right)^2-\left(\sqrt{3}-\sqrt{5}\right)^2\Big)$

$=\Big(3+2\sqrt{15}+5-7\Big)\Big(7-\left(3-2\sqrt{15}+5\right)\Big)$

$=\left(2\sqrt{15}+1\right)\left(2\sqrt{15}-1\right)=\left(2\sqrt{15}\right)^2-1^2=60-1=\underline{\underline{59}}$

(f) $\frac{0.2\sqrt{5}}{0.5\sqrt{2}}=\frac{\frac{2\sqrt{5}}{10}}{\frac{5\sqrt{2}}{10}}=\frac{2\sqrt{5}}{5\sqrt{2}}=\frac{2\sqrt{10}}{10}=\underline{\underline{\frac{\sqrt{10}}{5}}}$

(g) $\frac{\sqrt{2}-2\sqrt{3}}{\sqrt{6}}=\frac{\sqrt{12}-2\sqrt{18}}{6}=\frac{2\sqrt{3}}{6}-\frac{6\sqrt{2}}{6}=\underline{\underline{-\sqrt{2}+\frac{\sqrt{3}}{3}}}$

(h) $\frac{3}{4-\sqrt{7}}=\frac{3}{4-\sqrt{7}}\cdot\frac{4+\sqrt{7}}{4+\sqrt{7}}=\frac{12+3\sqrt{7}}{16-7}=\frac{12+3\sqrt{7}}{9}=\underline{\underline{\frac{4}{3}+\frac{\sqrt{7}}{3}}}$

(i) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\cdot\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{5-3}=\frac{5+2\sqrt{15}+3}{2}=\underline{\underline{4+\sqrt{15}}}$

(j) Hier kommen wir zum Ziel, indem wir ebenfalls den "Erweiterungs-Trick" anwenden:

$$\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}} = \frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}} \cdot \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{\sqrt{2}+\sqrt{3}-\sqrt{5}} = \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{\left(\sqrt{2}+\sqrt{3}\right)^2-\left(\sqrt{5}\right)^2}$$
$$= \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2+2\sqrt{6}+3-5} = \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{12}+\sqrt{18}-\sqrt{30}}{12}$$
$$= \frac{2\sqrt{3}}{12} + \frac{3\sqrt{2}}{12} - \frac{\sqrt{30}}{12} = \underline{\underline{\frac{\sqrt{2}}{4} + \frac{\sqrt{3}}{6} - \frac{\sqrt{30}}{12}}}$$

(k) $$\frac{\sqrt{3}}{1+\sqrt{3}} + \frac{2-\sqrt{3}}{\sqrt{3}} + \frac{1}{3-\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}+1} + \frac{2-\sqrt{3}}{\sqrt{3}} + \frac{1}{\sqrt{3}\left(\sqrt{3}-1\right)}$$
$$= \frac{\sqrt{3}\cdot\sqrt{3}\left(\sqrt{3}-1\right) + \left(2-\sqrt{3}\right)\cdot\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right) + \sqrt{3}+1}{\sqrt{3}\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}$$
$$= \frac{3\sqrt{3}-3+\left(2-\sqrt{3}\right)\cdot(3-1)+\sqrt{3}+1}{\sqrt{3}(3-1)}$$
$$= \frac{4\sqrt{3}-2+4-2\sqrt{3}}{2\sqrt{3}} = \frac{2\sqrt{3}+2}{2\sqrt{3}} = 1+\frac{1}{\sqrt{3}} = \underline{\underline{1+\frac{\sqrt{3}}{3}}}$$

(l) $$\frac{a}{\sqrt{a}-\sqrt{b}} = \frac{a}{\sqrt{a}-\sqrt{b}} \cdot \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}} = \frac{a\sqrt{a}+a\sqrt{b}}{a-b} = \underline{\underline{\frac{a\sqrt{a}}{a-b} + \frac{a\sqrt{b}}{a-b}}}$$

(m) $$\frac{2+3\sqrt{x}}{3+2\sqrt{x}} = \frac{2+3\sqrt{x}}{3+2\sqrt{x}} \cdot \frac{3-2\sqrt{x}}{3-2\sqrt{x}} = \frac{6+5\sqrt{x}-6x}{9-4x} = \underline{\underline{\frac{6(1-x)}{9-4x} + \frac{5\sqrt{x}}{9-4x}}}$$

(n) $$\frac{n}{\sqrt{n}} = \frac{\sqrt{n}\cdot\sqrt{n}}{\sqrt{n}} = \underline{\underline{\sqrt{n}}}$$

(o) $$\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} \cdot \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{x-2\sqrt{xy}+y}{x-y} = \underline{\underline{\frac{x+y}{x-y} - \frac{2\sqrt{xy}}{x-y}}}$$

(p) $$\frac{2\sqrt{xy}}{\sqrt{x}+\sqrt{y}+\sqrt{x+y}} = \frac{2\sqrt{xy}}{\sqrt{x}+\sqrt{y}+\sqrt{x+y}} \cdot \frac{\sqrt{x}+\sqrt{y}-\sqrt{x+y}}{\sqrt{x}+\sqrt{y}-\sqrt{x+y}}$$
$$= \frac{2\sqrt{xy}\left(\sqrt{x}+\sqrt{y}-\sqrt{x+y}\right)}{\left(\sqrt{x}+\sqrt{y}\right)^2-\left(\sqrt{x+y}\right)^2} = \frac{2\sqrt{xy}\left(\sqrt{x}+\sqrt{y}-\sqrt{x+y}\right)}{x+2\sqrt{xy}+y-x-y}$$
$$= \frac{2\sqrt{xy}\left(\sqrt{x}+\sqrt{y}-\sqrt{x+y}\right)}{2\sqrt{xy}} = \underline{\underline{\sqrt{x}+\sqrt{y}-\sqrt{x+y}}}$$

(q) $$\sqrt{\frac{1}{\frac{1}{c^2}+\frac{1}{d^2}-\frac{2}{cd}}} \cdot \left(\left(\frac{c-d}{cd}\right)^2 - \frac{2}{c^2} + \frac{2}{cd}\right) = \sqrt{\frac{1}{\frac{d^2+c^2-2cd}{c^2d^2}}} \cdot \frac{(c-d)^2-2d^2+2cd}{c^2d^2}$$
$$= \sqrt{\frac{c^2d^2}{d^2+c^2-2cd}} \cdot \frac{c^2-2cd+d^2-2d^2+2cd}{c^2d^2} = \sqrt{\frac{c^2d^2}{(c-d)^2}} \cdot \frac{c^2-d^2}{c^2d^2}$$
$$= \frac{|cd|}{|c-d|} \cdot \frac{(c+d)(c-d)}{c^2d^2} = \begin{cases} \frac{c+d}{|cd|} & \text{für } c>d \\ -\frac{c+d}{|cd|} & \text{für } c<d \end{cases}$$