

# Algebra – Lösungen zu Übungsserie 4

1. (a)  $(1\frac{1}{4} - \frac{3}{8}) : (\frac{1}{-2} + 1.25) = (\frac{5}{4} - \frac{3}{8}) : (-\frac{1}{2} + \frac{5}{4}) = \frac{10-3}{8} : \frac{-2+5}{4} = \frac{7}{8} : \frac{3}{4} = \frac{7}{8} \cdot \frac{4}{3} = \frac{7}{6}$
- (b)  $((\frac{4}{12})^3 + 2 : 3^3) : (\frac{1}{12} - \frac{1}{6} : 6) = ((\frac{1}{3})^3 + \frac{2}{3^3}) : (\frac{1}{12} - \frac{1}{36}) = \frac{1+2}{3^3} : \frac{3-1}{36} = \frac{3}{3^3} : \frac{2}{36} = \frac{3}{3^3} \cdot \frac{36}{2} = 2$
- (c)  $((\frac{4}{3})^2 - 1)(1 + \frac{2^2}{21} : \frac{2}{3}) = (\frac{16}{9} - \frac{9}{9})(1 + \frac{2}{7}) = \frac{7}{9} \cdot \frac{9}{7} = 1$
- (d)  $(\frac{7}{9} - 0.8 \cdot \frac{5}{12})(\frac{-4}{3} : 2 + 5 : 3 : 4) = (\frac{7}{9} - \frac{4}{5} \cdot \frac{5}{12})(-\frac{2}{3} + \frac{5}{12}) = (\frac{7}{9} - \frac{1}{3}) \cdot \frac{-8+5}{12} = \frac{4}{9} \cdot \frac{-3}{12} = -\frac{1}{9}$
- (e)  $(\frac{2}{3} \cdot \frac{6}{21} + 0.07) : (2.25 - 1.5^2) = (\frac{4}{21} + \frac{7}{100}) : (\frac{9}{4} - (\frac{3}{2})^2) = \frac{400+147}{2100} : (\frac{9}{4} - \frac{9}{4}) = 547 : 0$   
 $\Rightarrow$  Division durch 0 ist verboten  $\Rightarrow$  keine Lösung!
- (f)  $((1.875 + \frac{9}{8}) : (\frac{5}{5} - 1\frac{7}{8} - \frac{3}{8} \cdot 11))^4 = ((\frac{15}{8} + \frac{9}{8}) : (1 - \frac{15}{8} - \frac{33}{8}))^4 = (\frac{24}{8} \cdot \frac{-8}{40})^4 = (-\frac{3}{5})^4 = \frac{81}{625}$
2. (a)  $\frac{5}{8} = 0.625$
- (b)  $\frac{5}{14} = 0.3571428$
- (c)  $\frac{37}{125} = 0.296$
- (d)  $\frac{516}{3125} = 0.16512$
- (e)  $\frac{\sqrt{10}}{\sqrt{169}} = \frac{\sqrt{10}}{13} \Rightarrow \sqrt{10}$  ist keine rationale Zahl  $\rightarrow$  geht nicht!
3. (a)  $\frac{2^3+3^3}{2^2-2\cdot 3+3^2} = \frac{8+27}{4-6+9} = \frac{35}{7} = 5 = 2 + 3$
- (b)  $\frac{3^3+2^3}{3^2-3\cdot 2+2^2} = \frac{27+8}{9-6+4} = \frac{35}{7} = 5 = 3 + 2$
- (c)  $\frac{0^3+6^3}{0^2-0\cdot 6+6^2} = \frac{6^3}{6^2} = 6 = 0 + 6$
- (d)  $\frac{(-1)^3+4^3}{(-1)^2-(-1)\cdot 4+4^2} = \frac{-1+64}{1+4+16} = \frac{63}{21} = 3 = -1 + 4$
- (e)  $\frac{(\frac{1}{2})^3 + 1^3}{(\frac{1}{2})^2 - \frac{1}{2} \cdot 1 + 1^2} = \frac{\frac{1}{8}+1}{\frac{1}{4}-\frac{1}{2}+1} = \frac{\frac{9}{8}}{\frac{3}{4}} = \frac{9}{8} \cdot \frac{4}{3} = \frac{3}{2} = \frac{1}{2} + 1$
- (f)  $\frac{(\frac{2}{3})^3 + (\frac{1}{3})^3}{(\frac{2}{3})^2 - \frac{2}{3} \cdot \frac{1}{3} + (\frac{1}{3})^2} = \frac{\frac{8}{27} + \frac{1}{27}}{\frac{4}{9} - \frac{2}{9} + \frac{1}{9}} = \frac{\frac{9}{27}}{\frac{3}{9}} = \frac{9}{27} \cdot \frac{9}{3} = 1 = \frac{2}{3} + \frac{1}{3}$

**Vermutung:** Die Lösung ist stets gleich der Summe der beiden Zahlen  $a$  und  $b$ :

$$\frac{a^3 + b^3}{a^2 - ab + b^2} = a + b$$

**Beweis:** Das müsste sich beweisen lassen, indem wir die Gleichung mit dem Nenner multiplizieren und schauen, was auf der rechten Seite herauskommt:

$$(a + b) \cdot (a^2 - ab + b^2) = a^3 - a^2b + ab^2 + ba^2 - ab^2 + b^3 = a^3 + b^3 \Rightarrow \checkmark$$

**Ausnahme:** Einzig für  $a = b = 0$  ist die Beziehung falsch, denn in diesem Fall ist der Bruch gar nicht definiert, weil sein Nenner gleich Null ist.

4. (a)  $1.375 = \frac{11}{8}$
- (b)  $0.36 = \frac{36}{100} = \frac{9}{25}$
- (c)  $x = 0.\overline{36} \Rightarrow 100x = 36.\overline{36} \Rightarrow 99x = 36 \Rightarrow x = \frac{36}{99} = \frac{4}{11}$
- (d)  $1.288 = \frac{1288}{1000} = \frac{161}{125}$
- (e)  $x = 1.\overline{288} \Rightarrow 1000x = 1288.\overline{288} \Rightarrow 999x = 1287 \Rightarrow x = \frac{1287}{999} = \frac{143}{111}$
- (f)  $x = 0.\overline{2277} \Rightarrow 10\,000x = 2277.\overline{2277} \Rightarrow 9999x = 2277 \Rightarrow x = \frac{2277}{9999} = \frac{253}{1111} = \frac{23}{101}$

5. (a)  $(2 \cdot \frac{3}{5} - 2.6)(\frac{2}{-9} - 0.\overline{3}) = (\frac{6}{5} - \frac{13}{5})(-\frac{2}{9} - \frac{1}{3}) = -\frac{7}{5} \cdot \frac{-2-3}{9} = \frac{-7}{5} \cdot \frac{-5}{9} = \frac{7}{9}$
- (b)  $((4 + \frac{12}{5} - \frac{1}{20}) : 6.35 - 1)^5 = (\frac{80+48-1}{20} : \frac{635}{100} - 1)^5 = (\frac{127}{20} \cdot \frac{100}{635} - 1)^5$   
 $= (\frac{127}{20} \cdot \frac{20}{127} - 1)^5 = (1 - 1)^5 = 0^5 = 0$
- (c)  $(\frac{22}{3} : 2 + (-3)^2 : 3)^0 = 1$
- (d)  $((\frac{13}{12})^2 - (\frac{5}{12})^2) \cdot ((\frac{6}{7})^2 + (\frac{6}{7})^2 + (\frac{6}{7})^2) = \frac{169-25}{144} \cdot \frac{36+9+4}{49} = \frac{144}{144} \cdot \frac{49}{49} = 1$
- (e)  $x = 0.\overline{18} \Rightarrow 100x = 18.\overline{18} \Rightarrow 99x = 18 \Rightarrow x = \frac{18}{99} = \frac{2}{11}$   
 $(\frac{13}{1001} - 0.\overline{18})(\frac{(-2)^3}{-6^2-3} + \frac{1}{13}) = (\frac{1}{7 \cdot 11} - \frac{2}{11})(\frac{-8}{-39} + \frac{1}{13}) = \frac{1-14}{7 \cdot 11}(\frac{8}{39} + \frac{3}{39}) = \frac{-13}{7 \cdot 11} \cdot \frac{11}{39} = -\frac{1}{21}$
- (f)  $x = 0.8\overline{3} \Rightarrow 10x = 8.3\overline{3} \Rightarrow 9x = 7.5 \Rightarrow x = \frac{7.5}{9} = \frac{15}{18} = \frac{5}{6}$   
 $x = 0.1\overline{6} \Rightarrow 10x = 1.6\overline{6} \Rightarrow 9x = 1.5 \Rightarrow x = \frac{1.5}{9} = \frac{6}{36} = \frac{1}{6}$   
 $(0.8\overline{3} - \frac{1}{6}) : (2^3 : 5) : (\frac{7}{12} - 0.1\overline{6}) = (\frac{5}{6} - \frac{1}{6}) : \frac{8}{5} : (\frac{7}{12} - \frac{1}{6}) = \frac{4}{6} \cdot \frac{5}{8} : \frac{7-2}{12} = \frac{5}{3 \cdot 4} \cdot \frac{12}{5} = 1$