

Algebra – Lösungen zu Übungsserie 7

1. $\frac{25}{5x+10} = \frac{25}{5(x+2)} = \frac{5}{\underline{\underline{x+2}}}$
2. $\frac{2a}{4ac-2a} = \frac{2a}{2a(2c-1)} = \frac{1}{\underline{\underline{2c-1}}}$
3. $\frac{-b}{b^2+b} = -\frac{b}{b(b+1)} = -\frac{1}{\underline{\underline{b+1}}}$
4. $\frac{18a^2bc}{18a^2b^2c+54a^2bc^2} = \frac{18a^2bc}{18a^2bc(b+3c)} = \frac{1}{\underline{\underline{b+3c}}}$
5. $\frac{7a+14}{7a-21} = \frac{7(a+2)}{7(a-3)} = \frac{a+2}{\underline{\underline{a-3}}}$
6. $\frac{ab-ac}{ad-ac} = \frac{a(b-c)}{a(d-c)} = \frac{b-c}{\underline{\underline{d+c}}}$
7. $\frac{a^2-a}{ab+a} = \frac{a(a-1)}{a(b+1)} = \frac{a-1}{\underline{\underline{b+1}}}$
8. $\frac{3a+3b}{4a+4b} = \frac{3(a+b)}{4(a+b)} = \frac{3}{\underline{\underline{4}}}$
9. $\frac{2x-2}{5-5x} = \frac{2(x-1)}{5(1-x)} = \frac{2(x-1)}{-5(x-1)} = \underline{\underline{-\frac{2}{5}}}$
10. $\frac{rs+rt}{sx+tx} = \frac{r(s+t)}{x(s+t)} = \frac{r}{\underline{\underline{x}}}$
11. $\frac{4x^3-5x}{8x^2-10} = \frac{x(4x^2-5)}{2(x^2-5)} = \frac{x}{\underline{\underline{2}}}$
12. $\frac{4a^2-4a+1}{10a-5} = \frac{(2a-1)^2}{5(2a-1)} = \frac{2a-1}{\underline{\underline{5}}}$
13. $\frac{4a^2-20ab+25b^2}{2ac-5bc} = \frac{(2a-5b)^2}{c(2a-5b)} = \frac{2a-5b}{\underline{\underline{c}}}$
14. $\frac{x^3-2x^2+x}{2x-2} = \frac{x(x^2-2x+1)}{2(x-1)} = \frac{x(x-1)^2}{2(x-1)} = \frac{x(x-1)}{\underline{\underline{2}}}$
15. $\frac{x^4-2x^2+1}{x^3-x} = \frac{(x^2-1)^2}{x(x^2-1)} = \frac{x^2-1}{x} = \frac{(x+1)(x-1)}{\underline{\underline{x}}}$
16. $\frac{4x^2-1}{4x+2} = \frac{(2x+1)(2x-1)}{2(2x+1)} = \frac{2x-1}{\underline{\underline{2}}}$
17. $\frac{5x-5z}{x^2-z^2} = \frac{5(x-z)}{(x+z)(x-z)} = \frac{5}{\underline{\underline{x+z}}}$
18. $\frac{a^4-1}{a^2+1} = \frac{(a^2+1)(a^2-1)}{a^2+1} = a^2-1 = \underline{\underline{(a+1)(a-1)}}$
19. $\frac{x^2-y^2}{x^4-y^4} = \frac{x^2-y^2}{(x^2+y^2)(x^2-y^2)} = \frac{1}{\underline{\underline{x^2+y^2}}}$
20. $\frac{4a^2+25}{16a^4-625} = \frac{4a^2+25}{(4a^2+25)(4a^2-25)} = \frac{1}{4a^2-25} = \frac{1}{\underline{\underline{(2a+5)(2a-5)}}$

$$21. \frac{a^2 - b^2}{a^2 - 2ab + b^2} = \frac{(a+b)(a-b)}{(a-b)^2} = \frac{a+b}{\underline{\underline{a-b}}}$$

$$22. \frac{x^2 - 8x + 7}{2x^2 - 4x + 2} = \frac{(x-1)(x-7)}{2(x^2 - 2x + 1)} = \frac{(x-1)(x-7)}{2(x-1)^2} = \frac{x-7}{\underline{\underline{2(x-1)}}}$$

$$23. \frac{4x^2 - 4x + cx - c}{5x - 5} = \frac{4x(x-1) + c(x-1)}{5(x-1)} = \frac{(4x+c)(x-1)}{5(x-1)} = \frac{4x+c}{\underline{\underline{5}}}$$

$$24. \frac{9a^2 - 9ab}{a^2 - ab + ac - bc} = \frac{9a(a-b)}{a(a-b) + c(a-b)} = \frac{9a(a-b)}{(a+c)(a-b)} = \frac{9a}{\underline{\underline{a+c}}}$$

$$25. \frac{5am + 7an - 10bm - 14bn}{3a - 6b} = \frac{a(5m + 7n) - 2b(5m + 7n)}{2(a - 2b)} = \frac{(a - 2b)(5m + 7n)}{2(a - 2b)} = \frac{5m + 7n}{\underline{\underline{3}}}$$

$$26. \frac{a^2 - b^2 + 4a + 4b}{a^2 + 2ab + b^2} = \frac{(a+b)(a-b) + 4(a+b)}{(a+b)^2} = \frac{(a+b)(a-b+4)}{(a+b)^2} = \frac{a-b+4}{\underline{\underline{a+b}}}$$

$$27. \frac{a^2 + 2a - 15}{a^2 - 25} = \frac{(a+5)(a-3)}{(a+5)(a-5)} = \frac{a-3}{\underline{\underline{a-5}}}$$

$$28. \frac{144a^3 - 60a^2c - 156a^2 + 65ac}{12abc + 12ac^2 - 5bc^2 - 5c^3} = \frac{a(144a^2 - 60ac - 156a + 65c)}{c(12ab + 12ac - 5bc - 5c^2)}$$

$$= \frac{a(12a(12a - 5c) - 13(12a - 5c))}{c(12a(b+c) - 5c(b+c))} = \frac{a(12a - 13)(12a - 5c)}{c(12a - 5)(b+c)} = \frac{a(12a - 13)}{\underline{\underline{c(b+c)}}}$$

$$29. \frac{a^2 - 9a + 20}{a^2 - 10a + 25} = \frac{(a-4)(a-5)}{(a-5)^2} = \frac{a-4}{\underline{\underline{a-5}}}$$

$$30. \frac{a^4 - 1}{a^4 + 6a^2 + 5} = \frac{(a^2+1)(a^2-1)}{(a^2+5)(a^2+1)} = \frac{(a^2+1)(a+1)(a-1)}{(a^2+5)(a^2+1)} = \frac{(a+1)(a-1)}{\underline{\underline{a^2+5}}}$$

$$31. \frac{a^2 - 9ab + 20b^2}{a^2 - 2ab - 8b^2} = \frac{(a-4b)(a-5b)}{(a+2b)(a-4b)} = \frac{a-5b}{\underline{\underline{a+2b}}}$$

$$32. \frac{x^2 - 7x + 12}{2x^2 - x - 15} = \frac{(x-3)(x-4)}{(2x+5)(x-3)} = \frac{x-4}{\underline{\underline{2x+5}}}$$

$$33. \frac{20 - 33x^2 - 27x^4}{9x^2 + 12x + 4} = \frac{(4 - 9x^2)(5 + 3x^2)}{(3x+2)^2} = \frac{(2+3x)(2-3x)(5+3x^2)}{(3x+2)^2} = \frac{(2-3x)(5+3x^2)}{\underline{\underline{3x+2}}}$$

$$34. \frac{a-b}{b-a} = \frac{-(b-a)}{b-a} = \underline{\underline{-1}}$$

$$35. \frac{a-1}{1-a^2} = \frac{a-1}{(1+a)(1-a)} = \frac{-(1-a)}{(1+a)(1-a)} = \underline{\underline{-\frac{1}{a+1}}}$$

$$36. \frac{a^2 - 13a + 42}{14 - 2a} = \frac{(a-6)(a-7)}{2(7-a)} = \frac{-(a-6)(7-a)}{2(7-a)} = \frac{-(a-6)}{2} = \underline{\underline{\frac{6-a}{2}}}$$

$$37. \frac{80a^3 - 58a^2 + 11a - 3}{5a - 3} = \frac{(5a-3)(16a^2 - 2a + 1)}{5a - 3} = \underline{\underline{16a^2 - 2a + 1}}$$

Die Faktorisierung des Zählers in Aufgabe 37 ist anspruchsvoller. Der Nenner legt nahe: Man sollte versuchen $(5a - 3)$ auszuklammern. Während klar ist, wie das quadratische und das konstante Glied des Restausdrucks aussehen müssen, $\frac{80a^3}{5a} = 16a^2$ und $\frac{-3}{3} = 1$, müssen wir momentan beim linearen Glied noch ein bisschen "probieren". Dass $2a$ korrekt ist, wird durch Ausmultiplikation in die Gegenrichtung klar:

$$(5a - 3)(16a^2 - 2a + 1) = 80a^3 - 10a^2 + 5a - 48a^2 + 6a - 3 = 80a^3 - 58a^2 + 11a - 3$$

$$38. \frac{x-y}{x^3-y^3} = \frac{x-y}{(x-y)(x^2+xy+y^2)} = \frac{1}{\underline{\underline{x^2+xy+y^2}}}$$

$$39. \frac{m^5-n^5}{m-n} = \frac{(m-n)(m^4+m^3n+m^2n^2+mn^3+n^4)}{m-n} \\ = \frac{(m-n)(m^4+m^3n+m^2n^2+mn^3+n^4)}{m-n} = \underline{\underline{m^4+m^3n+m^2n^2+mn^3+n^4}}$$

Die Faktorisierungen in den Aufgaben 37, 39 und 40 gehören momentan nicht zu unseren Repertoire. Wir werden dafür später eine Technik erlernen. Aktuell kannst du dir selber zu überlegen versuchen, wie man sowas macht.

$$40. \frac{a^4+a^3-4a^2-5a-5}{a^2+a+1} = \frac{(a^2+a+1)(a^2-5)}{a^2+a+1} = \underline{\underline{a^2-5}} \left(= (a+\sqrt{5})(a-\sqrt{5}) \right)$$

$$41. \frac{a^2-b^2-2bc-c^2}{2a+2b+2c} = \frac{(a+b+c)(a-b-c)}{2(a+b+c)} = \underline{\underline{\frac{a-b-c}{2}}}$$

$$42. \frac{4a^2+10a+25}{8a^3-125} = \frac{4a^2+10a+25}{(2a-5)(4a^2+10a+25)} = \underline{\underline{\frac{1}{2a-5}}}$$

$$43. \frac{36m^2-60mn+25n^2-9p^2}{6m-5n-3p} = \frac{(6m-5n-3p)(6m-5n+3p)}{6m-5n-3p} = \underline{\underline{6m-5n+3p}}$$

$$44. \frac{4x^2-(2x+3y)^2}{16x^2+24xy+9y^2} = \frac{(2x+(2x+3y))(2x-(2x+3y))}{(4x+3y)^2} = \frac{(4x+3y) \cdot (-3y)}{(4x+3y)^2} = \underline{\underline{-\frac{3y}{4x+3y}}}$$

$$45. (2x-1)^5 = (2x)^5 - 5(2x)^4 + 10(2x)^3 - 10(2x)^2 + 5 \cdot 2x - 1 \\ = \underline{\underline{32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x - 1}}$$

$$46. (3a^2-4b^3)(9a^4+12a^2b^3+16b^6) = (3a^2)^3 - (4b^3)^3 = \underline{\underline{27a^6 - 64b^9}}$$

$$47. (3p^2+2q^3)^4 = (3p^2)^4 + 4(3p^2)^3 \cdot 2q^3 + 6(3p^2)^2(2q^3)^2 + 4 \cdot 3p^2(2q^3)^3 + (2q^3)^4 \\ = \underline{\underline{81p^8 + 216p^6q^3 + 216p^4q^6 + 96p^2q^9 + 16q^{12}}}$$

$$48. (12c+3d+7e)(12c-3d+7e) = ((12c+7e)+3d)((12c+7e)-3d) \\ = \underline{\underline{144c^2 + 168ce + 49e^2 - 9d^2}}$$