

Algebra Zusatztraining – Lösungen

Faktorzerlegung von Polynomen

Ausklammern

200. (a) $7e - 7 = \underline{7(e - 1)}$ (b) $2rs + s = \underline{s(2r + 1)}$ (c) $p^3 + p^2 = \underline{p^2(p + 1)}$
(d) $36uvw + 9uw = \underline{9uw(4v + 1)}$
201. (a) $14f - 21g + 28 = \underline{7(2f - 3g + 4)}$ (b) $10at + 15bt - 6ct = \underline{t(10a + 15b - 6c)}$
(c) $xy - y^2 - yz = \underline{y(x - y - z)}$
202. (a) $15x - 27y - 12z = \underline{3(5x - 9y - 4z)}$ (b) $13r + 65s - 91 = \underline{13(r + 5s - 7)}$
(c) $14np - 12nq + 21n = \underline{n(14p - 12q + 21)}$
203. (a) $18a^2b + 18ab^2 - 9ab = \underline{9ab(2a + 2b - 1)}$ (b) $4x^2yz - 10xy^2z + 16xyz^2 = \underline{2xyz(2x - 5y + 8z)}$
204. (a) $42m^3n^2 - 70m^2n^3 - 42m^2n^2 = \underline{14m^2n^2(3m - 5n - 3)}$
(b) $3qr^2 + 3r^3 + 3r^2s - r^2 = \underline{r^2(3q + 3r + 3s - 1)}$
205. (a) $-y - 2 = \underline{-(y + 2)}$ (b) $-5c + d = \underline{-(5c - d)}$ (c) $-3m + 4n - 1 = \underline{-(3m - 4n + 1)}$
(d) $u - v - w = \underline{-(-u + v + w)}$ (e) $-7x^2 + 4x + 11 = \underline{-(7x^2 - 4x - 11)}$
(f) $-a_1 - a_2 + a_3 - a_4 = \underline{-(a_1 + a_2 - a_3 + a_4)}$
206. (a) $-mx + q = \underline{-(mx - q)}$ (b) $-6r - s - 8t = \underline{-(6r + s + 8t)}$
(c) $-c^2 - d^2 + 36 = \underline{-(c^2 + d^2 - 36)}$ (d) $-b_1 + b_2 - b_3 + b_4 = \underline{-(b_1 - b_2 + b_3 - b_4)}$
(e) $z^5 - z^4 - z^3 + z^2 - z - 1 = \underline{-(-z^5 + z^4 + z^3 - z^2 + z + 1)}$
207. (a) $2n + \frac{4}{5} = \underline{2(n + \frac{2}{5})}$ (b) $4u + 3v + 2w = \underline{2(2u + \frac{3}{2}v + w)}$ (c) $2a - \frac{5}{4}b + \frac{6}{7} = \underline{2(a - \frac{5}{8}b + \frac{3}{7})}$
208. (a) $3p - 4 = \underline{3(p - \frac{4}{3})}$ (b) $3x^2 + \frac{9}{4}x - \frac{3}{4} = \underline{3(x^2 + \frac{3}{4}x - \frac{1}{4})}$
(c) $\frac{1}{2}rs - \frac{2}{3}r - \frac{3}{4}s = \underline{3(\frac{1}{6}rs - \frac{2}{9}r - \frac{1}{4}s)}$
209. (a) $\frac{1}{6}a + \frac{3}{2}b = \underline{\frac{1}{6}(a + 9b)}$ (b) $\frac{1}{2}q^2 - q + \frac{2}{3} = \underline{\frac{1}{6}(3q^2 - 6q + 4)}$ (c) $4c + 5d - \frac{1}{6} = \underline{\frac{1}{6}(24c + 30d - 1)}$
210. (a) $-1.2t^2 + 3.6 = \underline{-1.2(t^2 - 3)}$ (b) $6e - 2.4f - 8.4 = \underline{-1.2(-5e + 2f + 7)}$
(c) $-1.2x + y + 1.5z = \underline{-1.2(x - \frac{5}{6}y - \frac{5}{4}z)}$
211. (a) $(a+b)(4a+4b) = 4(a+b)^2 = \underline{4a^2 + 8ab + 4b^2}$ (b) $(2n-2)(3n-3) = 6(n-1)^2 = \underline{6n^2 - 12n + 6}$
(c) $(1.5u - 1.5v)(6u + 6v) = 9(u - v)(u + v) = \underline{9u^2 - 9v^2}$
212. (a) $(7f - 7g)(f - g) = 7(f - g)^2 = \underline{7f^2 - 14fg + 7g^2}$
(b) $(5r + 5s)(8r - 8s) = 40(r + s)(r - s) = \underline{40r^2 - 40s^2}$
(c) $(2.5c + 2.5)(0.4c + 0.4) = (c + 1)^2 = \underline{c^2 + 2c + 1}$
213. (a) $(9xy + 9y) : (x + 1) = 9y(x + 1) : (x + 1) = \underline{9y}$
(b) $(4.5ac - 7.5ad) = 1.5a(3c - 5d) : (3c - 5d) = \underline{1.5a}$
214. (a) $(18ab - 12b^2) : (3a - 2b) = 6b(3a - 2b) : 3a - 2b = \underline{6b}$
(b) $(0.7x^2y + 2.8xy^2) : (x + 4y) = 0.7xy(x + 4y) : (x + 4y) = \underline{0.7xy}$

215. (a) $(a + 2)x + (b - 3)x = x(a + 2 + b - 3) = \underline{\underline{x(a + b - 1)}}$
 (b) $r(2u + 3v) - r(u + v) = r(2u + 3v - u - v) = \underline{\underline{r(u + 2v)}}$
 (c) $e^2(n - 4) - e^2(2n - 7) = e^2(n - 4 - 2n + 7) = \underline{\underline{e^2(-n + 3)}}$
 (d) $(p^2 - 5p)z + (p^2 + p)z = pz(p - 5 + p + 1) = pz(2p - 4) = \underline{\underline{2pz(p - 2)}}$
216. (a) $(8 - t)y - (6 - 2t)y = y(8 - t - 6 + 2t) = \underline{\underline{y(2 + t)}}$
 (b) $(3a - 5b)x + (b + 1)x = x(3a - 5b + b + 1) = \underline{\underline{x(3a - 4b + 1)}}$
 (c) $(2k^3 - k^2)r - (k^3 - 7k^2)r = k^2r(2k - 1 - k + 7) = \underline{\underline{k^2r(k + 6)}}$
 (d) $d^2(e - f + g) - d^2(f + g - h) = d^2(e - f + g - f - g + h) = \underline{\underline{d^2(e - 2f + h)}}$
217. (a) $a(x + y) + b(x + y) = \underline{\underline{(a + b)(x + y)}}$ (b) $m(u + v) - 3(u + v) = \underline{\underline{(m - 3)(u + v)}}$
 (c) $cd(6c - d) - 4(6c - d) = \underline{\underline{(cd - 4)(6c - d)}}$ (d) $q(r - s) + (r - s) = \underline{\underline{(q + 1)(r - s)}}$
218. (a) $(m + n)y + (m + n)z = \underline{\underline{(m + n)(y + z)}}$ (b) $2a(a - b) - b(a - b) = \underline{\underline{(2a - b)(a - b)}}$
 (c) $(5e - 1) - c(5e - 1) = \underline{\underline{(5e - 1)(1 - c)}}$ (d) $(f + g) - d(f + g) = \underline{\underline{(1 - d)(f + g)}}$
219. (a) $5p(3p - 2) + (-3p + 2) = \underline{\underline{(5p - 1)(3p - 2)}}$ (b) $x(y - z) - (z - y) = \underline{\underline{(x + 1)(y - z)}}$
220. (a) $s(st - 4) + t(4 - st) = \underline{\underline{(s - t)(st - 4)}}$ (b) $r(-r + 2) + (r - 2) = \underline{\underline{(1 - r)(r - 2)}}$
221. (a) $4x(a + b) - 5y(a + b) - 6(a + b) - 3x(a + b) - (a + b) = \underline{\underline{(a + b)(x - 5y - 7)}}$
 (b) $3p^2(u - v) - 2p(v - u) - 8(u - v) + (u - v) = \underline{\underline{(u - v)(3p^2 + 2p - 7)}}$
222. (a) $-a(x - y) + 2b(x - y) - 3c(x - y) + 4(x - y) = \underline{\underline{(x - y)(4 - a - 2b - 3c)}}$
 (b) $7m(r + s) - 3n(r + s) - 4(r + s) - n(r + s) + (r + s) = \underline{\underline{(r + s)(7m - 4n - 3)}}$
223. (a) $4v(p + q) - 8w(p + q) = \underline{\underline{4(p + q)(v - 2w)}}$ (b) $(t^2 - t)z + 9(t^2 - t) = \underline{\underline{t(t - 1)(z + 9)}}$
 (c) $a^3(2ab - c) + a^2(2ab - c) = \underline{\underline{a^2(a + 1)(2ab - c)}}$ (d) $10q(9e - 6) - 5(9e - 6) = \underline{\underline{15(2q - 1)(3e - 2)}}$
224. (a) $3w(2u - 6v) + 5(2u - 6v) = \underline{\underline{2(3w + 5)(u - 3v)}}$ (b) $2r^2(r - 7) - r(-r + 7) = \underline{\underline{r(2r + 1)(r - 7)}}$
 (c) $a^2(xy + xz - x) - ab(xy + xz - x) + a(xy + xz - x) = \underline{\underline{ax(a - b + 1)(y + z - 1)}}$
225. (a) $(e - 4f)(f + g) + 2e(f + g) = \underline{\underline{(f + g)(3e - 4f)}}$ (b) $(c - d)(n + 5) + (c - d)(2n + 3) = \underline{\underline{(c - d)(3n + 8)}}$
 (c) $q(2x - 3y) - (q + 1)(-2x + 3y) = \underline{\underline{(2q + 1)(2x - 3y)}}$
 (d) $(3a - 5c)(m + 4) - (a + c)(m + 4) = (2a - 6c)(m + 4) = \underline{\underline{2(a - 3c)(m + 4)}}$
226. (a) $8p(2p - 5) - (2p + 5)(2p - 5) = \underline{\underline{(6p - 5)(2p - 5)}}$
 (b) $(c^3 + c^2)(r - 3) + (c^3 + c^2)(r - 1) = c^2(c + 1)(2r - 4) = \underline{\underline{2c^2(c + 1)(r - 2)}}$
 (c) $(3u + v)(u - v) - (3u + v)(u - w) = \underline{\underline{(3u + v)(w - v)}}$
 (d) $(a - b)(5z - 1) + (2b - 2a)(z + 4) = (a - b)(5z - 1 + 2z + 8) = (a - b)(7z + 7) = \underline{\underline{7(a - b)(z + 1)}}$

Ausklammern in Teilsommen

227. (a) $a(x + y) + 2x + 2y = \underline{\underline{(a + 2)(x + y)}}$ (b) $bq + cq - (b + c)r = \underline{\underline{(q - r)(b + c)}}$
 (c) $2u - v + 5r(2u - v) = \underline{\underline{(5r + 1)(2u - v)}}$ (d) $7k(4n - 3) - 4n + 3 = \underline{\underline{(4n - 3)(7k - 1)}}$
228. (a) $a(3a - 2b) + 9ac - 6bc = a(3a - 2b) + 3c(3a - 2b) = \underline{\underline{(a + 3c)(3a - 2b)}}$
 (b) $4m(p + q) - p - q = \underline{\underline{(4m - 1)(p + q)}}$ (c) $(t - 5)x - ty + 5y = \underline{\underline{(t - 5)(x - y)}}$
 (d) $r^2 - r + (r - 1)s = r(r - 1) + (r - 1)s = \underline{\underline{(r + s)(r - 1)}}$

229. (a) $au + av + bu + bv = a(u + v) + b(u + v) = \underline{\underline{(a + b)(u + v)}}$
 (b) $j^2 - jk + 2j - 2k = j(j - k) + 2(j - k) = \underline{\underline{(j + 2)(j - k)}}$
 (c) $-2cx + cy - 4dx + 2dy = c(y - 2x) + 2d(y - 2x) = \underline{\underline{(c + 2d)(y - 2x)}}$
 (d) $12st + 16s - 27t - 36 = 4s(3t + 4) - 9(3t + 4) = \underline{\underline{(4s - 9)(3t + 4)}}$
 (e) $24pz - 39p - 16qz + 26q = 3p(8z - 13) - 2q(8z - 13) = \underline{\underline{(3p - 2q)(8z - 13)}}$
 (f) $35f^2 - 63fg - 15f + 27g = 7f(5f - 9g) - 3(5f - 9g) = \underline{\underline{(7f - 3)(5f - 9g)}}$
230. (a) $81ab + 72ad + 36bc + 32cd = 9a(9b + 8d) + 4c(9b + 8d) = \underline{\underline{(9a + 4c)(9b + 8d)}}$
 (b) $mn - m + n - 1 = m(n - 1) - (n - 1) = \underline{\underline{(m - 1)(n - 1)}}$
 (c) $8v^2 - 2vw - 12v + 3w = 2v(4v - w) - 3(4v - w) = \underline{\underline{(2v - 3)(4v - w)}}$
 (d) $20xy - 15xz - 24y + 18z = 5x(4y - 3z) - 6(4y - 3z) = \underline{\underline{(5x - 6)(4y - 3z)}}$
 (e) $20r^2s + 4rs^2 - 5r - s = 4rs(5r + s) - (5r + s) = \underline{\underline{(4rs - 1)(5r + s)}}$
 (f) $-21ef - 56eg + 6fg + 16g^2 = -7e(3f + 8g) + 2g(3f + 8g) = \underline{\underline{(2g - 7e)(3f + 8g)}}$
231. (a) $4amx + 4amy + 4anx + 4any = 4am(x + y) + 4an(x + y) = \underline{\underline{4a(m + n)(x + y)}}$
 (b) $6ab + 3a - 12b - 6 = 3a(2b + 1) - 6(2b + 1) = \underline{\underline{3(a - 2)(2b + 1)}}$
 (c) $u^4 - u^3v - 2u^3w + 2u^2vw = u^3(u - v) - 2u^2w(u - v) = \underline{\underline{u^2(u - 2w)(u - v)}}$
 (d) $40r^3s^2 - 60r^2s^2 + 16r^3s = 20r^2s^2(2r - 3) + 8r^2s(2r - 3) = \underline{\underline{4r^2s(5s + 2)(2r - 3)}}$
232. (a) $5act - 20adt + 15bct - 60bdt = 5at(c - 4d) + 15bt(c - 4d) = \underline{\underline{5t(a + 3b)(c - 4d)}}$
 (b) $-e^2fg - ef^2g + efg^2 + f^2g^2 = -efg(e + f) + fg^2(e + f) = \underline{\underline{fg(g - e)(e + f)}}$
 (c) $28pq - 42p - 24q + 36 = 14p(2q - 3) - 12(2q - 3) = \underline{\underline{2(7p - 6)(2q - 3)}}$
 (d) $-18x^2y^2 + 36xy^2z + 30xy^3 - 60y^3z = 18xy^2(-x + 2z) + 30y^3(x - 2z) = \underline{\underline{6y^2(3x - 5y)(2z - x)}}$
233. (a) $mx + my + mz + nx + ny + nz = \underline{\underline{(m + n)(x + y + z)}}$
 (b) $as + at + bs + bt + cs + ct = \underline{\underline{(a + b + c)(s + t)}}$
 (c) $eu + fu - ev - fv + ew + fw = \underline{\underline{(e + f)(u - v + w)}}$
 (d) $3kp + 3kq - 3kr - 6p - 6q + 6r = \underline{\underline{3(k - 2)(p + q - r)}}$
234. (a) $ar - a + br - b + cr - c = \underline{\underline{(a + b + c)(r - 1)}}$
 (b) $efm - efn - ef + egm - egn - eg = e(f + g)(m - n - 1)$
 (c) $-px - py - pz + 5x + 5y + 5z = \underline{\underline{(5 - p)(x + y + z)}}$
 (d) $u^2 - uv - uw - u + v + w = \underline{\underline{(u - 1)(u - v - w)}}$
235. (a) $2a^2 + 10ab - 12ac + 5a + 25b - 30c = \underline{\underline{(2a + 5)(a + 5b - 6c)}}$
 (b) $2pr^2 + 4pr - 3qr^2 - 6qr - r^2 - 2r = r(2p - 3q - 1)(r + 2)$
236. (a) $22cv - 22ct + 66c - 33dv + 33dt - 99d = \underline{\underline{11(2c - 3d)(v - t + 3)}}$
 (b) $15mnx - 5mny + 10mnz - 3x + y - 2z = \underline{\underline{(5mn - 1)(3x - y + 2z)}}$

Faktorzerlegung mit Hilfe von Formeln

237. (a) $x^2 - y^2 = \underline{\underline{(x + y)(x - y)}}$ (b) $4c^2 - 9d^2 = \underline{\underline{(2c + 3d)(2c - 3d)}}$ (c) $z^2 - 225 = \underline{\underline{(z + 15)(z - 15)}}$
 (d) $36n^2 - 1 = \underline{\underline{(6n + 1)(6n - 1)}}$ (e) $-a^2 + 324b^2 = \underline{\underline{(18b + a)(18b - a)}}$
 (f) $-u^2v^2 + 1 = \underline{\underline{(1 + uv)(1 - uv)}}$ (g) $16p^2 - q^4 = \underline{\underline{(4p + q^2)(4p - q^2)}}$
 (h) $x^4 - y^4 = \underline{\underline{(x^2 + y^2)(x^2 - y^2)}}$ $\underline{\underline{(x^2 + y^2)(x + y)(x - y)}}$

238. (a) $16m^2 - 9n^2 = \underline{(4m + 3n)(4m - 3n)}$ (b) $25x^2 - 1 = \underline{(5x + 1)(5x - 1)}$
(c) $-4s^2 + 49t^2 = \underline{(7t + 2s)(7t - 2s)}$ (d) $121q^2 - 576 = \underline{(11q + 24)(11q - 24)}$
(e) $u^2v^2 - 64w^2 = \underline{(uv + 8w)(uv - 8w)}$ (f) $-p^2 + 289 = \underline{(17 + p)(17 - p)}$
(g) $r^4 - 1 = (r^2 + 1)(r^2 - 1) = \underline{(r^2 + 1)(r + 1)(r - 1)}$ (h) $-y^4z^2 + 81 = \underline{(9 + y^2z)(9 - y^2z)}$
239. (a) $6a^2 - 6b^2 = \underline{6(a + b)(a - b)}$ (b) $9k^4 - 36k^2 = 9k^2(k^2 - 4) = \underline{9k^2(k + 2)(k - 2)}$
(c) $n^3 - n = n(n^2 - 1) = \underline{n(n + 1)(n - 1)}$ (d) $-50e^2 + 338 = 2(169 - 25e^2) = \underline{2(13 + 5e)(13 - 5e)}$
240. (a) $18z^2 - 2 = \underline{2(3z + 1)(3z - 1)}$ (b) $75r^2 - 147 = \underline{3(5r + 7)(5r - 7)}$
(c) $-c^4d^2 + 4c^2 = c^2(4 - c^2d^2) = \underline{c^2(2 + cd)(2 - cd)}$
(d) $x^6y^4 - x^2y^8 = x^2y^4(x^4 - y^4) = x^2y^4(x^2 + y^2)(x^2 - y^2) = \underline{x^2y^4(x^2 + y^2)(x + y)(x - y)}$
241. (a) $a(x^2 - y^2) + b(x^2 - y^2) = (a + b)(x^2 - y^2) = \underline{(a + b)(x + y)(x - y)}$
(b) $p^2u + 2p^2v - 4u - 8v = (p^2 - 4)(u + 2v) = \underline{(p + 2)(p - 2)(u + 2v)}$
242. (a) $63km^2 - 28kn^2 + 45m^2 - 20n^2 = (7k + 5)(9m^2 - 4n^2) = \underline{(7k + 5)(3m + 2n)(3m - 2n)}$
(b) $cr^2 - c - dr^2 + d = (c - d)(r^2 - 1) = \underline{(c - d)(r + 1)(r - 1)}$
243. (a) $x^2 - 2xy + y^2 = \underline{(x - y)^2}$ (b) $36u^2 + 60uv + 25v^2 = \underline{(6u + 5v)^2}$ (c) $n^2 - 4n + 4 = \underline{(n - 2)^2}$
(d) $4c^2 + 28cd + 49d^2 = \underline{(2c + 7d)^2}$ (e) $9q^2 - 6q + 1 = \underline{(3q - 1)^2}$
(f) $a^4 - 2a^2b^2 + b^4 = (a^2 - b^2)^2 = ((a + b)(a - b))^2 = \underline{(a + b)^2(a - b)^2}$
244. (a) $m^2 - 2m + 1 = \underline{(m - 1)^2}$ (b) $4f^2 - 20fg + 25g^2 = \underline{(2f - 5g)^2}$ (c) $x^2 + 16x + 64 = \underline{(x + 8)^2}$
(d) $16r^2 - 24rs + 9s^2 = \underline{(4r - 3s)^2}$ (e) $p^4 - 8p^2 + 16 = (p^2 - 4)^2 = \underline{(p + 2)^2(p - 2)^2}$
(f) $a^4 - 2a^2b^2 + b^4 = (a^2 - b^2)^2 = \underline{(a + b)^2(a - b)^2}$
245. (a) $5a^2 - 10ab + 5b^2 = 5(a^2 - 2ab + b^2) = \underline{5(a - b)^2}$ (b) $xy^2 + 2xy + x = \underline{x(y + 1)^2}$
(c) $-3u^2 + 18uv - 27v^2 = \underline{-3(u - 3v)^2}$
246. (a) $72n^2 + 168n + 98 = 2(36n^2 + 84n + 49) = \underline{2(6n + 7)^2}$
(b) $-q^2r^2 + 4qr - 4 = -(q^2r^2 - 4qr + 4) = \underline{-(qr - 2)^2}$
(c) $-c^4 - 2c^3d - c^2d^2 = -c^2(c^2 + 2cd + d^2) = \underline{-c^2(c + d)^2}$
247. (a) $a^2 + 2ab + b^2 - 36z^2 = (a + b)^2 - (6z)^2 = \underline{(a + b + 6z)(a + b - 6z)}$
(b) $p^2 - x^2 - 2x - 1 = p^2 - (x + 1)^2 = (p + (x + 1))(p - (x + 1)) = \underline{(p + x + 1)(p - x - 1)}$
248. (a) $u^2 - 8uv + 16v^2 - 1 = (u - 4v)^2 - 1^2 = \underline{(u - 4v + 1)(u - 4v - 1)}$
(b) $m^2 - q^2 + 10q - 25 = m^2 - (q - 5)^2 = (m + (q - 5))(m - (q - 5)) = \underline{(m + q - 5)(m - q + 5)}$

Klammeransatz bei geeigneten Trinomen

249. (a) $x^2 + 9x + 20 = \underline{(x + 4)(x + 5)}$ (b) $d^2 + 20d + 91 = \underline{(d + 7)(d + 13)}$
(c) $r^2 - 15r + 54 = \underline{(r - 6)(r - 9)}$ (d) $n^2 - 26n + 144 = \underline{(n - 8)(n - 18)}$
(e) $n^2 - 24n + 144 = \underline{(n - 12)^2}$ (f) $3c^2 + 16c + 5 = \underline{(3c + 1)(c + 5)}$
250. (a) $s^2 + 18s + 72 = \underline{(s + 8)(s + 9)}$ (b) $z^2 - 19z + 48 = \underline{(z - 16)(z - 3)}$
(c) $p^2 + 23p + 132 = \underline{(p + 11)(p + 12)}$ (d) $y^2 - 29y + 210 = \underline{(y - 14)(y - 15)}$
(e) $b^2 + 10b + 9 = \underline{(b + 1)(b + 9)}$ (f) $2x^2 - 5x + 2 = \underline{(2x - 1)(x - 2)}$

251. (a) $a^2 + 2a - 24 = \underline{(a + 6)(a - 4)}$ (b) $u^2 - 3u - 40 = \underline{(u + 5)(u - 8)}$
 (c) $t^2 - 6t - 7 = \underline{(t + 1)(t - 7)}$ (d) $x^2 - 25x + 84 = \underline{(x - 4)(x - 21)}$
 (e) $x^2 + 25x - 84 = \underline{(x + 28)(x - 3)}$ (f) $4e^2 + 3e - 1 = \underline{(4e - 1)(e + 1)}$
252. (a) $c^2 - 3c - 108 = \underline{(c + 9)(c - 12)}$ (b) $m^2 + 4m - 5 = \underline{(m + 5)(m - 1)}$
 (c) $y^2 - y - 30 = \underline{(y + 5)(y - 6)}$ (d) $z^2 + 9z - 90 = \underline{(z + 15)(z - 6)}$
 (e) $r^2 - 43r - 240 = \underline{(r + 5)(r - 48)}$ (f) $5k^2 - 2k - 3 = \underline{(5k + 3)(k - 1)}$
253. (a) $b^2 + 20b + 51 = \underline{(b + 3)(b + 17)}$ (b) $t^2 + t - 156 = \underline{(t + 13)(t - 12)}$
 (c) $x^2 - 4x + 16$ lässt sich gar nicht faktorisieren! (d) $v^2 - 7v - 98 = \underline{(v + 7)(v - 14)}$
 (e) $p^2 - 7p - 120 = \underline{(p + 8)(p - 15)}$ (f) $2n^2 + 7n + 3 = \underline{(2n + 1)(n + 3)}$
254. (a) $m^2 - m - 110 = \underline{(m + 10)(m - 11)}$ (b) $z^2 - 29z + 208 = \underline{(z - 13)(z - 16)}$
 (c) $q^2 - 16q - 36 = \underline{(q + 2)(q - 18)}$ (d) $y^2 + 40y + 400 = \underline{(y + 20)^2}$
 (e) $a^2 + 6a - 10$ lässt sich faktorisieren, aber nicht mit einem klassischen Zweiklammeransatz!
 (f) $12r^2 - 8r + 1 = \underline{(6r - 1)(2r - 1)}$
255. (a) $5x^2 + 10x - 75 = 5(x^2 + 2x - 15) = \underline{5(x + 5)(x - 3)}$
 (b) $n^3 - n^2 - n = \underline{n(n^2 - n - 1)}$ lässt sich weiter faktorisieren, aber nicht mit einem klassischen Zweiklammeransatz!
 (c) $-4t^2 - 4t + 48 = -4(t^2 + t - 12) = \underline{-4(t + 4)(t - 3)}$
256. (a) $9z^4 - 36z^3 + 27z^2 = 9z^2(z^2 - 4z + 3) = \underline{9z^2(z - 1)(z - 3)}$
 (b) $-3k^2 - 3k - 60 = \underline{-3(k^2 + k + 20)}$ lässt sich nicht weiter faktorisieren!
 (c) $2b^5 + 9b^4 - 5b^3 = b^3(2b^2 + 9b - 5) = \underline{b^3(2b - 1)(b + 5)}$
257. (a) $x^2 - 7xy + 10y^2 = \underline{(x - 2y)(x - 5y)}$ (b) $p^2 - 2pq - 8q^2 = \underline{(p + 2q)(p - 4q)}$
 (c) $m^4 - 5m^2n - 24n^2 = \underline{(m^2 + 3n)(m^2 - 8n)}$
258. (a) $a^2 + 5ab + 4b^2 = \underline{(a + 4b)(a + b)}$ (b) $r^2 + 4rs - 21s^2 = \underline{(r + 7s)(r - 3s)}$
 (c) $c^4 - 13c^2d^2 + 36d^4 = (c^2 - 4d^2)(c^2 - 9d^2) = \underline{(c + 2d)(c - 2d)(c + 3d)(c - 3d)}$

Vermischte und schwierigere Aufgaben zur Faktorzerlegung

259. (a) $-16x^5 + x = x(1 - 16x^4) = x(1 + 4x^2)(1 - 4x^2) = \underline{x(1 + 4x^2)(1 + 2x)(1 - 2x)}$
 (b) $n^3 - 19n^2 + 90n = n(n^2 - 19n + 90) = \underline{n(n - 9)(n - 10)}$
 (c) $fgh + fg + fh + f = f(gh + g + h + 1) = f(g(h + 1) + (h + 1)) = \underline{f(g + 1)(h + 1)}$
260. (a) $625c^3 - 225cd^2 = 25c(25c^2 - 9d^2) = \underline{25c(5c + 3d)(5c - 3d)}$
 (b) $-3z^4 + 6z^3 + 24z^2 = -3z^2(z^2 - 2z - 8) = \underline{-3z^2(z + 2)(z - 4)}$
 (c) $64st - 48s - 48t + 36 = 4(16st - 12s - 12t + 9) = 4(4s(4t - 3) - 3(4t - 3)) = \underline{4(4s - 3)(4t - 3)}$

Kürzen und Erweitern (von Bruchtermen)

Erweitern

45. (a) $\frac{6}{7x} \cdot \frac{12yz}{12yz} = \frac{72yz}{84xyz}$ (b) $\frac{3u}{8} \cdot \frac{3u}{3u} = \frac{9u^2}{24u}$ (c) $\frac{a+b}{ab^2} \cdot \frac{a^2b}{a^2b} = \frac{a^3b+a^2b^2}{a^3b^3}$ (d) $2r \cdot \frac{3}{3} = \frac{6r}{3}$

(e) $\frac{15w}{10-w} \cdot \frac{-w}{-w} = \frac{-15w^2}{w^2-10w}$ (f) $p^2 \cdot \frac{p(p-1)}{p(p-1)} = \frac{p^3(p-1)}{p^2-p}$ (g) $\frac{s-t}{s+t} \cdot \frac{s-t}{s-t} = \frac{(s-t)^2}{s^2-t^2}$

46. (a) $\frac{4ac}{3b} \cdot \frac{4bc^2}{4bc^2} = \frac{16abc^3}{12b^2c^2}$ (b) $\frac{4q}{3q+2} \cdot \frac{2q}{2q} = \frac{8q^2}{6q^2+4q}$ (c) $7 \cdot \frac{2w}{2w} = \frac{14w}{2w}$ (d) $k \cdot \frac{k}{k} = \frac{k^2}{k}$

(e) $\frac{6mn}{2m-5n} \cdot \frac{4n}{4n} = \frac{24mn^2}{8mn-20n^2}$ (f) $\frac{-2}{4y-3x} \cdot \frac{-2}{-2} = \frac{4}{6x-8y}$ (g) $\frac{d}{2d+1} \cdot \frac{9(2d+1)}{9(2d+1)} = \frac{18d^2+9d}{(6d+3)^2}$

47. (a) $\frac{2}{a}, \frac{3}{b}, \frac{4}{c} \rightarrow \frac{2bc}{abc}, \frac{3ac}{abc}, \frac{4ab}{abc}$ (b) $\frac{7}{8w}, \frac{5}{6w} \rightarrow \frac{21}{24w}, \frac{20}{24w}$ (c) $\frac{p}{e^2}, \frac{p}{e^3} \rightarrow \frac{pe}{e^3}, \frac{p}{e^3}$

(d) $\frac{r^2}{9s^2u}, \frac{1}{r^2u^2}, \frac{8u}{15rs} \rightarrow \frac{5r^4u}{45r^2s^2u^2}, \frac{45s^2}{45r^2s^2u^2}, \frac{24rsu^3}{45r^2s^2u^2}$

48. (a) $\frac{u}{2v}, \frac{v}{2u} \rightarrow \frac{u^2}{2uv}, \frac{v^2}{2uv}$ (b) $\frac{x}{yz}, \frac{y}{xz}, \frac{z}{xy} \rightarrow \frac{x^2}{xyz}, \frac{y^2}{xyz}, \frac{z^2}{xyz}$ (c) $\frac{15}{4mn^2}, \frac{25}{6m^3n} \rightarrow \frac{45m^2}{12m^3n^2}, \frac{50n}{12m^3n^2}$

(d) $\frac{13}{6h^2}, 1, \frac{4}{21hi} \rightarrow \frac{91i}{42h^2i}, \frac{42h^2i}{42h^2i}, \frac{8h}{42h^2i}$

49. (a) $\frac{1}{rs}, \frac{1}{r^2+r} \rightarrow \frac{r+1}{rs(r+1)}, \frac{s}{rs(r+1)}$ (b) $\frac{a}{b}, \frac{a}{b+c} \rightarrow \frac{a(b+c)}{b(b+c)}, \frac{ab}{b(b+c)}$

(c) $\frac{q}{q^2-1}, \frac{q-1}{q+1} \rightarrow \frac{q}{(q+1)(q-1)}, \frac{(q-1)^2}{(q+1)(q-1)}$

50. (a) $\frac{x}{x+y}, \frac{y}{x-y} \rightarrow \frac{x(x-y)}{(x+y)(x-y)}, \frac{y(x+y)}{(x+y)(x-y)}$ (b) $\frac{1}{t^2-t}, \frac{t-1}{t} \rightarrow \frac{1}{t(t-1)}, \frac{(t-1)^2}{t(t-1)}$

(c) $\frac{3}{uv+v}, \frac{u+v}{2v} \rightarrow \frac{6}{2v(u+1)}, \frac{(u+1)(u+v)}{2v(u+1)}$

51. (a) $\frac{n}{n-5}, \frac{5}{5-n} \rightarrow \frac{n}{n-5}, \frac{-5}{n-5}$ (b) $\frac{w-z}{w+z}, \frac{w+z}{w-z} \rightarrow \frac{(w-z)^2}{(w+z)(w-z)}, \frac{(w+z)^2}{(w+z)(w-z)}$

(c) $\frac{a}{a^2-b^2}, \frac{b}{b-a} \rightarrow \frac{a}{(a+b)(a-b)}, \frac{-b(a+b)}{(a+b)(a-b)}$

52. (a) $\frac{3c+2d}{3c-2d}, \frac{2c+3d}{2c-3d} \rightarrow \frac{(3c+2d)(2c-3d)}{(3c-2d)(2c-3d)}, \frac{(2c+3d)(3c-2d)}{(3c-2d)(2c-3d)}$ (b) $\frac{21}{2x-2}, \frac{-31}{3x-3}, \frac{41}{4x-4} \rightarrow \frac{126}{12(x-1)}, \frac{-124}{12(x-1)}, \frac{123}{12(x-1)}$

53. Mache zuerst gleichnamig: $\frac{a}{b} = \frac{a(b+1)}{b(b+1)} = \frac{ab+a}{b(b+1)}$ und $\frac{a+1}{b+1} = \frac{b(a+1)}{b(b+1)} = \frac{ab+b}{b(b+1)}$.

Nun kann man die Zähler vergleichen: $ab + a \geq ab + b \Leftrightarrow a \geq b$

Somit ist $\frac{a}{b}$ genau dann größer als $\frac{a+1}{b+1}$, wenn $a > b$ ist.

Beispiel zur Verdeutlichung: $\frac{3}{2} > \frac{3+1}{2+1} = \frac{4}{3}$, aber $\frac{2}{3} < \frac{2+1}{3+1} = \frac{3}{4}$.

54. Mache wieder gleichnamig: $\frac{a}{b} = \frac{a(b+d)}{b(b+d)} = \frac{ab+ad}{b(b+d)}$ und $\frac{a+d}{b+d} = \frac{b(a+d)}{b(b+d)} = \frac{ab+bd}{b(b+d)}$.

Nun kann man die Zähler vergleichen: $ab + ad \geq ab + bd \Leftrightarrow ad \geq bd \Leftrightarrow a \geq b$ (geht, weil $d > 0$)

Somit ist $\frac{a}{b}$ wieder genau dann größer als $\frac{a+d}{b+d}$, wenn $a > b$ ist.

Beispiele zur Verdeutlichung: $\frac{3}{2} > \frac{3+4}{2+4} = \frac{7}{6}$, aber $\frac{2}{3} < \frac{2+4}{3+4} = \frac{6}{7}$.

Addition und Subtraktion von Bruchtermen

55. (a) $\frac{2x}{3} + \frac{4x}{3} = \frac{6x}{3} = \underline{\underline{2x}}$ (b) $\frac{7}{8a} - \frac{1}{8a} = \frac{6}{8a} = \underline{\underline{\frac{3}{4a}}}$ (c) $\frac{5}{3n} + \frac{2}{3n} - \frac{-5}{3n} = \frac{12}{3n} = \underline{\underline{\frac{4}{n}}}$

56. (a) $\frac{5z}{6} - \frac{z}{6} = \frac{4z}{6} = \underline{\underline{\frac{2z}{3}}}$ (b) $\frac{5c}{12y} + \frac{c}{12y} = \frac{6c}{12y} = \underline{\underline{\frac{c}{2y}}}$ (c) $\frac{-76u}{35v} - \frac{8u}{35v} = \frac{-84u}{35v} = \underline{\underline{\frac{12u}{5v}}}$

57. (a) $\frac{a+b}{2} + \frac{a-b}{2} = \frac{2a}{2} = \underline{\underline{a}}$ (b) $\frac{a+nb}{n} - \frac{a-nb}{n} = \frac{2nb}{n} = \underline{\underline{2b}}$ (c) $-\frac{3r+4}{6} + \frac{5r+7}{6} = \underline{\underline{\frac{2r+3}{6}}}$

58. (a) $\frac{-t+7}{4t} - \frac{3t+4}{4t} - \frac{8t-5}{4t} = \frac{-12t+12}{4t} = \underline{\underline{\frac{-3t+3}{t}}}$ (b) $\frac{x^2+x-8}{2x} - \frac{x^2-7x-3}{2x} + \frac{2x^2-4x+5}{2x} = \frac{2x^2+4x}{2x} = \underline{\underline{x+2}}$
59. (a) $\frac{1}{m+1} + \frac{m}{m+1} = \frac{1+m}{m+1} = \underline{\underline{1}}$ (b) $\frac{cd}{b-d} - \frac{bc}{b-d} = \frac{c(d-b)}{b-d} = \underline{\underline{-c}}$
(c) $\frac{xy}{y^2-2yz+z^2} - \frac{xz}{y^2-2yz+z^2} = \frac{x(y-z)}{(y-z)^2} = \underline{\underline{\frac{x}{y-z}}}$ (d) $\frac{4a}{4a^2+7a+3} + \frac{3}{4a^2+7a+3} = \frac{4a+3}{(4a+3)(a+1)} = \underline{\underline{\frac{1}{a+1}}}$
60. (a) $\frac{q}{p-q} - \frac{p}{p-q} = \frac{q-p}{p-q} = \underline{\underline{-1}}$ (b) $\frac{4ktw}{2t-1} - \frac{2kw}{2t-1} = \frac{2kw(2t-1)}{2t-1} = \underline{\underline{2kw}}$
(c) $-\frac{s^3}{s^2-1} + \frac{s^2}{s^2-1} = \frac{s^2(1-s)}{(s+1)(s-1)} = \underline{\underline{-\frac{s^2}{s+1}}}$ (d) $\frac{52x}{65x^2+59x-72} - \frac{36}{65x^2+59x-72} = \frac{4(13x-9)}{(13x-9)(5x+8)} = \underline{\underline{\frac{4}{5x+8}}}$
61. (a) $\frac{e}{2} - \frac{e}{3} = \frac{3e-2e}{6} = \underline{\underline{\frac{e}{6}}}$ (b) $\frac{2p}{15q} + \frac{8p}{9q} = \frac{6p+40p}{45q} = \underline{\underline{\frac{46p}{45q}}}$ (c) $\frac{5}{6ac} - \frac{3}{4cd} = \underline{\underline{\frac{10d-9a}{12acd}}}$
(d) $\frac{1}{r^2} - \frac{1}{r^3} = \underline{\underline{\frac{r-1}{r^3}}}$
62. (a) $\frac{4u}{21} + \frac{9u}{14} = \frac{8u+27u}{42} = \frac{35u}{42} = \underline{\underline{\frac{5u}{6}}}$ (b) $\frac{8}{9m} - \frac{11}{36m} = \frac{32-11}{36m} = \frac{21}{36m} = \underline{\underline{\frac{7}{12m}}}$ (c) $\frac{z}{n^2} + \frac{4}{3n} = \underline{\underline{\frac{3z+4n}{3n^2}}}$
(d) $\frac{7v}{10w} - \frac{5v}{6w} = \frac{21v-25v}{30w} = \frac{-4v}{30w} = \underline{\underline{-\frac{2v}{15w}}}$
63. (a) $\frac{7s}{18} - \frac{4s-9}{45} = \frac{35s-8s+18}{90} = \frac{27s+18}{90} = \underline{\underline{\frac{3s+2}{10}}}$ (b) $\frac{a+b}{b} - \frac{a-b}{a} = \frac{a(a+b)-b(a-b)}{ab} = \frac{a^2+ab-ab+b^2}{ab} = \underline{\underline{\frac{a^2+b^2}{ab}}}$
(c) $\frac{x+y}{2xy} + \frac{x+z}{2xz} + \frac{y+z}{2yz} = \frac{z(x+y)+y(x+z)+x(y+z)}{2xyz} = \frac{2xy+2xz+2yz}{2xyz} = \underline{\underline{\frac{xy+xz+yz}{xyz}}}$
64. (a) $\frac{c-2}{c^3} + \frac{c-1}{c^2} = \frac{c-2+c^2-c}{c^3} = \underline{\underline{\frac{c^2-2}{c^3}}}$
(b) $\frac{(u-v)^2}{u^2v^2} - \frac{2u+v}{u^2v} + \frac{u-3v}{uv^2} = \frac{u^2-2uv+v^2-2uv-v^2+u^2-3uv}{u^2v^2} = \frac{2u^2-7uv}{u^2v^2} = \underline{\underline{\frac{2u-7v}{uv^2}}}$
65. (a) $\frac{a}{3} + 1 = \underline{\underline{\frac{a+3}{3}}}$ (b) $7r - \frac{9}{2s} = \underline{\underline{\frac{14rs-9}{2s}}}$ (c) $5w - 1 + \frac{3}{w} = \underline{\underline{\frac{5w^2-w+3}{w}}}$
66. (a) $8m - \frac{n}{5} = \underline{\underline{\frac{40m-n}{5}}}$ (b) $b + \frac{1}{b} = \underline{\underline{\frac{b^2+1}{b}}}$ (c) $\frac{x}{4z} - 2y + 3z = \underline{\underline{\frac{x-8yz+12z^2}{4z}}}$
67. (a) $\frac{2r+3}{6} + 1 = \frac{2r+3+6}{6} = \frac{2r+9}{6}$ (b) $t - 4 - \frac{t+1}{2} = \frac{2t-8-t-1}{2} = \frac{t-9}{2}$ (c) $d - \frac{nd-2}{n} = \frac{nd-nd+2}{n} = \underline{\underline{\frac{2}{n}}}$
68. (a) $p + \frac{9-p}{2} = \frac{2p+9-p}{2} = \frac{p+9}{2}$ (b) $\frac{x-y}{3x} - 1 = \frac{x-y-3x}{3x} = \underline{\underline{-\frac{2x+y}{3x}}}$
(c) $2 - \frac{k^2-k+1}{k^2} = \frac{2k^2-k^2+k-1}{k^2} = \underline{\underline{\frac{k^2+k-1}{k^2}}}$
69. (a) $\frac{2a}{a+b} + 1 = \frac{2a+a+b}{a+b} = \underline{\underline{\frac{3a+b}{a+b}}}$ (b) $4 - \frac{u-v}{u+v} = \frac{4u+4v-u+v}{u+v} = \underline{\underline{\frac{3u+5v}{u+v}}}$ (c) $\frac{z^2}{z+1} - z = \frac{z^2-z^2-z}{z+1} = \underline{\underline{-\frac{z}{z+1}}}$
70. (a) $3 - \frac{m}{m-n} = \frac{3m-3n-m}{m-n} = \frac{2m-3n}{m-n}$ (b) $\frac{q}{q+1} - 1 = \frac{q-q-1}{q+1} = \underline{\underline{-\frac{1}{q+1}}}$
(c) $e - \frac{e^2-2}{e-2} = \frac{e^2-2e-e^2+2}{e-2} = \frac{-2e+2}{e-2} = \underline{\underline{-\frac{2(e-1)}{e-2}}}$
(d) Das Resultat lautet stets $\underline{\underline{\frac{1}{1-z}}}$ (vgl. unsere Serie VIII, Aufgabe 13).
71. (a) $\frac{1}{a+b} + \frac{1}{c} = \frac{a+b+c}{c(a+b)}$ (b) $\frac{8}{n+5} - \frac{n+2}{n} = \frac{8n-(n+2)(n+5)}{n(n+5)} = \frac{8n-n^2-7n-10}{n(n+5)} = \frac{-n^2+n-10}{n(n+5)} = \underline{\underline{-\frac{n^2-n+10}{n(n+5)}}}$
(c) $\frac{x+y}{x-y} - \frac{x-y}{x+y} = \frac{(x+y)^2-(x-y)^2}{(x+y)(x-y)} = \frac{x^2+2xy+y^2-x^2+2xy-y^2}{(x+y)(x-y)} = \underline{\underline{\frac{4xy}{(x+y)(x-y)}}}$
72. (a) $\frac{m}{m-1} - \frac{m-1}{m+2} = \frac{m(m+2)-(m-1)^2}{(m-1)(m+2)} = \frac{m^2+2m-m^2+2m-1}{(m-1)(m+2)} = \underline{\underline{\frac{4m-1}{(m-1)(m+2)}}}$
(b) $\frac{2r}{s} - \frac{r+3}{r+s+1} = \frac{2r(r+s+1)-s(r+3)}{s(r+s+1)} = \frac{2r^2+2rs+2r-rs-3s}{s(r+s+1)} = \underline{\underline{\frac{2r^2+rs+2r-3s}{s(r+s+1)}}}$
(c) $\frac{w-4}{w-2} + \frac{w+6}{w+3} = \frac{(w-4)(w+3)+(w+6)(w-2)}{(w-2)(w+3)} = \frac{w^2-w-12+w^2+4w-12}{(w-2)(w+3)} = \underline{\underline{\frac{2w^2+3w-24}{(w-2)(w+3)}}}$
73. (a) $\frac{c}{c+d} - \frac{c-d}{2(c+d)} = \frac{2c-c+d}{2(c+d)} = \frac{c+d}{2(c+d)} = \underline{\underline{\frac{1}{2}}}$ (b) $\frac{4}{z-1} + \frac{z}{z^2-1} = \frac{4(z+1)+z}{(z+1)(z-1)} = \underline{\underline{\frac{5z+4}{(z+1)(z-1)}}}$
(c) $\frac{3u}{u^2+2uv+v^2} - \frac{1}{u+v} = \frac{3u-u-v}{(u+v)^2} = \underline{\underline{\frac{2u-v}{(u+v)^2}}}$ (d) $\frac{a+2b+t}{4at+8bt} - \frac{1}{4t} = \frac{a+2b+t-a-2b}{4t(a+2b)} = \frac{t}{4t(a+2b)} = \underline{\underline{\frac{1}{4(a+2b)}}}$

74. (a) $\frac{x-y}{15x+10y} + \frac{x+y}{3x+2y} = \frac{x-y+5(x+y)}{5(3x+2y)} = \frac{6x+4y}{5(3x+2y)} = \frac{2(3x+2y)}{5(3x+2y)} = \underline{\underline{\frac{2}{5}}}$
 (b) $\frac{8p}{4p^2-4p+1} - \frac{3}{2p-1} = \frac{8p-6p+3}{(2p-1)^2} = \frac{2p+3}{(2p-1)^2}$
 (c) $\frac{r+2}{5r^2} - \frac{4r+4}{5r^3+10r^2} = \frac{(r+2)^2-4r-4}{5r^2(r+2)} = \frac{r^2+4r+4-4r-4}{5r^2(r+2)} = \frac{r^2}{5r^2(r+2)} = \underline{\underline{\frac{1}{5(r+2)}}}$
 (d) $\frac{1}{q-1} - \frac{q^2+2}{q^3-1} = \frac{q^2+q+1-q^2-2}{(q-1)(q^2+q+1)} = \frac{q-1}{(q-1)(q^2+q+1)} = \underline{\underline{\frac{1}{q^2+q+1}}}$ (Kuben-Formel!)
75. (a) $\frac{c}{c-d} - \frac{2cd}{c^2-d^2} - \frac{d}{c+d} = \frac{c^2+cd-2cd-cd+d^2}{(c+d)(c-d)} = \frac{c^2-2cd+d^2}{(c+d)(c-d)} = \frac{(c-d)^2}{(c+d)(c-d)} = \underline{\underline{\frac{c-d}{c+d}}}$
 (b) $\frac{1}{a-2} + \frac{1}{a+5} - \frac{2a+3}{a^2+3a-10} = \frac{a+5+a-2-2a-3}{(a+5)(a-2)} = \frac{0}{(a+5)(a-2)} = \underline{\underline{0}}$
76. (a) $\frac{z}{z-5} - \frac{5}{z+3} - \frac{40}{z^2-2z-15} = \frac{z^2+3z-5z+25-40}{(z+3)(z-5)} = \frac{z^2-2z-15}{(z+3)(z-5)} = \frac{(z+3)(z-5)}{(z+3)(z-5)} = \underline{\underline{1}}$
 (b) $\frac{n}{n+1} - \frac{2n+1}{n-1} + \frac{n^2+5n}{n^2-1} = \frac{n^2-1-2n^2-3n-1+n^2+5n}{(n+1)(n-1)} = \frac{2n-2}{(n+1)(n-1)} = \underline{\underline{\frac{2}{n+1}}}$
77. (a) $\frac{a-b}{4a+4b} + \frac{a+4b}{6a+6b} = \frac{3a-3b+2a+8b}{12(a+b)} = \frac{5a+5b}{12(a+b)} = \underline{\underline{\frac{5}{12}}}$
 (b) $\frac{t+7}{3t-6} - \frac{t+4}{t^2-2t} = \frac{t^2+7t-3t-12}{3t(t-2)} = \frac{t^2+4t-12}{3t(t-2)} = \frac{(t+6)(t-2)}{3t(t-2)} = \underline{\underline{\frac{t+6}{3t}}}$
 (c) $\frac{u}{uv+v^2} - \frac{v}{u^2+uv} = \frac{u^2-v^2}{uv(u+v)} = \frac{(u+v)(u-v)}{uv(u+v)} = \underline{\underline{\frac{u-v}{uv}}}$
 (d) $\frac{c}{c^2-8c+16} + \frac{2}{c^2-6c+8} = \frac{c}{(c-4)^2} + \frac{2}{(c-4)(c-2)} = \frac{c^2-2c+2c-8}{(c-2)(c-4)^2} = \underline{\underline{\frac{c^2-8}{(c-2)(c-4)^2}}}$
78. (a) $\frac{1}{rx+ry} + \frac{1}{sx+sy} = \frac{r+s}{rs(x+y)}$ (b) $\frac{a}{a^2-b^2} + \frac{b}{(a-b)^2} = \frac{a^2-ab+ab+b^2}{(a+b)(a-b)^2} = \underline{\underline{\frac{a^2+b^2}{(a+b)(a-b)^2}}}$
 (c) $\frac{z+9}{z^2-1} - \frac{z+5}{z^2+z} = \frac{z+9}{(z+1)(z-1)} - \frac{z+5}{z(z+1)} = \frac{z^2+9z-z^2-4z+5}{z(z+1)(z-1)} = \frac{5z+5}{z(z+1)(z-1)} = \underline{\underline{\frac{5}{z(z-1)}}}$
 (d) $\frac{5}{n^2+n-6} - \frac{3}{n^2-n-2} = \frac{5}{(n+3)(n-2)} - \frac{3}{(n+1)(n-2)} = \frac{5n+5-3n-9}{(n+3)(n+1)(n-2)} = \frac{2n-4}{(n+3)(n+1)(n-2)} = \underline{\underline{\frac{2}{(n+3)(n+1)}}}$
79. (a) $\frac{7}{e-1} + \frac{6}{1-e} = \frac{7-6}{e-1} = \underline{\underline{\frac{1}{e-1}}}$ (b) $\frac{5}{3h-3} - \frac{4}{2-2h} = \frac{5}{3(h-1)} - \frac{4}{2(1-h)} = \frac{5}{3(h-1)} + \frac{2}{h-1} = \frac{5+6}{3(h-1)} = \underline{\underline{\frac{11}{3(h-1)}}}$
 (c) $\frac{r-4}{5r+5} + \frac{2}{1-r^2} = \frac{r-4}{5(r+1)} - \frac{2}{(r+1)(r-1)} = \frac{(r-4)(r-1)-10}{5(r+1)(r-1)} = \frac{r^2-5r+4-10}{5(r+1)(r-1)} = \frac{r^2-5r-6}{5(r+1)(r-1)} = \frac{(r+1)(r-6)}{5(r+1)(r-1)} = \underline{\underline{\frac{r-6}{5(r-1)}}}$
 (d) $\frac{u}{u-v} - \frac{4uv}{u^2-v^2} - \frac{v}{v-u} = \frac{u(u+v)-4uv+v(u+v)}{(u+v)(u-v)} = \frac{u^2+uv-4uv+uv+v^2}{(u+v)(u-v)} = \frac{u^2-2uv+v^2}{(u+v)(u-v)} = \frac{(u-v)^2}{(u+v)(u-v)} = \underline{\underline{\frac{u-v}{u+v}}}$
80. (a) $\frac{a-b}{c-d} - \frac{a+b}{d-c} = \frac{a-b+a+b}{c-d} = \underline{\underline{\frac{2a}{c-d}}}$ (b) $\frac{x+y}{2x-6y} + \frac{x+3y}{9y-3x} = \frac{x+y}{2(x-3y)} - \frac{x+3y}{3(x-3y)} = \frac{3x+3y-2x-6y}{6(x-3y)} = \underline{\underline{\frac{1}{6}}}$
 (c) $\frac{8s}{s^2-4} + \frac{2+s}{2-s} = \frac{8s}{(s+2)(s-2)} - \frac{s+2}{s-2} = \frac{8s-s^2-4s-4}{(s+2)(s-2)} = \frac{-(s-2)^2}{(s+2)(s-2)} = \underline{\underline{-\frac{s-2}{s+2}}} = \underline{\underline{\frac{2-s}{s+2}}}$
 (d) $\frac{m^2-8m}{2m^2+m-15} - \frac{m}{5-2m} = \frac{m^2-8m}{(2m-5)(m+3)} + \frac{m}{2m-5} = \frac{m^2-8m+m^2+3m}{(2m-5)(m+3)} = \frac{2m^2-5m}{(2m-5)(m+3)} = \underline{\underline{\frac{m}{m+3}}}$
81. (a) $\frac{2n-11}{3n-5} - \frac{4n+15}{n+7} + 1 = \frac{(2n-11)(n+7)-(4n+15)(3n-5)+(3n-5)(n+7)}{(3n-5)(n+7)} = \frac{2n^2+3n-77-12n^2-25n+75+3n^2+16n-35}{(3n-5)(n+7)} = \frac{-7n^2-6n-37}{(3n-5)(n+7)} = \underline{\underline{-\frac{7n^2+6n+37}{(3n-5)(n+7)}}}$
 (b) $\frac{2v+3w}{2v+w} - \frac{2v-w}{2v} - \frac{2v+3w}{w} = \frac{2vw(2v+3w)-w(2v+w)(2v-w)-2v(2v+w)(2v+3w)}{2vw(2v+w)} = \frac{4v^2w+6vw^2-4v^2w+w^3-8v^3-16v^2w-6vw^2}{2vw(2v+w)} = \underline{\underline{-\frac{8v^3-16v^2w+w^3}{2vw(2v+w)}}}$
82. (a) $\frac{2r-19}{3r-7} - \frac{5r}{6r-8} - \frac{1}{2} = \frac{2(2r-19)(3r-4)-5r(3r-7)-(3r-7)(3r-4)}{2(3r-7)(3r-4)} = \frac{12r^2-130r+152-15r^2+35r-9r^2+33r-28}{2(3r-7)(3r-4)} = \frac{-12r^2-62r+124}{2(3r-7)(3r-4)} = \frac{-6r^2-31r+62}{(3r-7)(3r-4)} = \underline{\underline{-\frac{6r^2+31r-62}{(3r-7)(3r-4)}}}$
 (b) $\frac{5}{p-2} - \frac{3}{2p+1} + \frac{1}{p+1} = \frac{5(2p+1)(p+1)-3(p-2)(p+1)+(p-2)(2p+1)}{(p-2)(2p+1)(p+1)} = \frac{10p^2+15p+5-3p^2+3p+6+2p^2-3p-2}{(p-2)(2p+1)(p+1)} = \frac{9p^2+15p+9}{(p-2)(2p+1)(p+1)} = \underline{\underline{\frac{3(3p^2+5p+3)}{(p-2)(2p+1)(p+1)}}}$
83. (a) $\frac{5}{4x-8y} - \frac{3}{10y-5x} - \frac{11}{6x-12y} = \frac{5}{4(x-2y)} + \frac{3}{5(x-2y)} - \frac{11}{6(x-2y)} = \frac{75+36-110}{60(x-2y)} = \underline{\underline{\frac{1}{60(x-2y)}}}$
 (b) $\frac{b-c}{a^2+ac} - \frac{a-b}{ac+c^2} + \frac{a^2+c^2}{a^2c+ac^2} = \frac{c(b-c)-a(a-b)+a^2+c^2}{ac(a+c)} = \frac{bc-c^2-a^2+ab+a^2+c^2}{ac(a+c)} = \frac{bc+ab}{ac(a+c)} = \underline{\underline{\frac{b}{ac}}}$

$$84. \quad (a) \quad \frac{k+2}{6k-15} + \frac{8k+1}{8k-20} + \frac{k+11}{10-4k} = \frac{k+2}{3(2k-5)} + \frac{8k+1}{4(2k-5)} - \frac{k+11}{2(2k-5)} = \frac{4k+8+24k+3-6k-66}{12(2k-5)} = \frac{22k-55}{12(2k-5)} = \underline{\underline{\frac{11}{12}}}$$

$$(b) \quad \frac{u}{u-v} + \frac{v}{v-u} - \frac{u+v-1}{u+v} = \frac{u(u+v)-v(u+v)-(u+v-1)(u-v)}{(u+v)(u-v)} = \frac{u^2+uv-uv-v^2-u^2+uv-uv+v^2+u-v}{(u+v)(u-v)} \\ = \frac{u-v}{(u+v)(u-v)} = \underline{\underline{\frac{1}{u+v}}}$$

$$85. \quad (a) \quad \frac{2x-1}{x-3} - \frac{2x(x+2)}{x^2-9} - \frac{2}{3x} = \frac{3x(x+3)(2x-1)-6x^2(x+2)-2(x^2-9)}{3x(x+3)(x-3)} = \frac{6x^3+15x^2-9x-6x^3-12x^2-2x^2+18}{3x(x+3)(x-3)} \\ = \frac{x^2-9x+18}{3x(x+3)(x-3)} = \frac{(x-3)(x-6)}{3x(x+3)(x-3)} = \underline{\underline{\frac{x-6}{3x(x+3)}}}$$

$$(b) \quad \frac{3s}{(s-2)^2} - \frac{2}{s} + \frac{s+4}{2s-s^2} = \frac{3s^2-2(s-2)^2-(s+4)(s-2)}{s(s-2)^2} = \frac{3s^2-2s^2+8s-8-s^2-2s+8}{s(s-2)^2} = \frac{6s}{s(s-2)^2} = \underline{\underline{\frac{6}{(s-2)^2}}}$$

$$86. \quad (a) \quad \frac{2u-v}{2u-2v} - \frac{u-v}{3u+3v} - \frac{v(3v-u)}{3(v^2-u^2)} = \frac{3(u+v)(2u-v)-2(u-v)^2+2v(3v-u)}{6(u+v)(u-v)} = \frac{6u^2+3uv-3v^2-2u^2+4uv-2v^2+6v^2-2uv}{6(u+v)(u-v)} \\ = \frac{4u^2+5uv+v^2}{6(u+v)(u-v)} = \frac{(4u+v)(u+v)}{6(u+v)(u-v)} = \underline{\underline{\frac{4u+v}{6(u-v)}}}$$

$$(b) \quad \frac{1}{z^2-z} - \frac{2}{z^2} + \frac{1}{z^2+z} = \frac{z^2+z-2(z^2-1)+z^2-z}{z^2(z+1)(z-1)} = \frac{z^2-2z^2+2+z^2}{z^2(z+1)(z-1)} = \underline{\underline{\frac{2}{z^2(z+1)(z-1)}}}$$

$$87. \quad \frac{a}{(a-b)(a-c)} + \frac{b}{(b-c)(b-a)} + \frac{c}{(c-a)(c-b)} = \frac{a(b-c)-b(a-c)+c(a-b)}{(a-b)(a-c)(b-c)} = \frac{ab-ac-ab+bc+ac-bc}{(a-b)(a-c)(b-c)} = \underline{\underline{0}}$$

$$88. \quad \frac{x^4+36x^2-32}{x^4-8x^2+16} - \frac{16x}{x^3+2x^2-4x-8} - \frac{16x}{x^3-2x^2-4x+8} - 1 = \frac{x^4+36x^2-32}{(x^2-4)^2} - \frac{16x}{x^2(x+2)-4(x+2)} - \frac{16x}{x^2(x-2)-4(x-2)} - 1 \\ = \frac{x^4+36x^2-32}{(x+2)^2(x-2)^2} - \frac{16x}{(x^2-4)(x+2)} - \frac{16x}{(x^2-4)(x-2)} - 1 = \frac{x^4+36x^2-32}{(x+2)^2(x-2)^2} - \frac{16x}{(x+2)^2(x-2)} - \frac{16x}{(x+2)(x-2)^2} - 1 \\ = \frac{x^4+36x^2-32-16x(x-2)-16x(x+2)-(x+2)^2(x-2)^2}{(x+2)^2(x-2)^2} = \frac{x^4+36x^2-32-16x^2+32x-16x^2-32x-x^4+8x^2-16}{(x+2)^2(x-2)^2} \\ = \frac{12x^2-48}{(x+2)^2(x-2)^2} = \frac{12(x^2-4)}{(x+2)^2(x-2)^2} = \frac{12(x+2)(x-2)}{(x+2)^2(x-2)^2} = \underline{\underline{\frac{12}{(x+2)(x-2)}}}$$

$$89. \quad \frac{a^2+3a+5}{a^4-a^3-31a^2+25a+150} - \frac{a+2}{a^3-3a^2-25a+75} + \frac{a-3}{a^3+2a^2-25a-50} - \frac{a-5}{a^3+4a^2-11a-30} \\ = \frac{a^2+3a+5}{a^4-a^3-31a^2+25a+150} - \frac{a+2}{a^2(a-3)-25(a-3)} + \frac{a-3}{a^2(a+2)-25(a+2)} - \frac{a-5}{a^3+4a^2-11a-30} \\ = \frac{a^2+3a+5}{a^4-a^3-31a^2+25a+150} - \frac{a+2}{(a^2-25)(a-3)} + \frac{a-3}{(a^2-25)(a+2)} - \frac{a-5}{a^3+4a^2-11a-30} \\ = \frac{a^2+3a+5}{a^4-a^3-31a^2+25a+150} - \frac{a+2}{(a+5)(a-5)(a-3)} + \frac{a-3}{(a+5)(a-5)(a+2)} - \frac{a-5}{a^3+4a^2-11a-30} = (*)$$

Dank diesen beiden gelungenen Nennerfaktorierungen kann man es nun bei den anderen Brüchen ebenfalls probieren. In Frage kommen die Faktoren $(a \pm 5)$ und $(a \pm 2)$. Wir finden:

$$a^4 - a^3 - 31a^2 + 25a + 150 = (a+5)(a^3 - 6a^2 - a + 30) = (a+5)(a-5)(a^2 - a - 6) \\ = (a+5)(a-5)(a+2)(a-3)$$

$$a^3 + 4a^2 - 11a - 30 = (a+5)(a^2 - a - 6) = (a+5)(a+2)(a-3) \quad \text{Damit folgt:}$$

$$(*) = \frac{a^2+3a+5}{(a+5)(a-5)(a+2)(a-3)} - \frac{a+2}{(a+5)(a-5)(a-3)} + \frac{a-3}{(a+5)(a-5)(a+2)} - \frac{a-5}{(a+5)(a+2)(a-3)} \\ = \frac{a^2+3a+5-(a+2)^2+(a-3)^2-(a-5)^2}{(a+5)(a-5)(a+2)(a-3)} = \frac{a^2+3a+5-a^2-4a-4+a^2-6a+9-a^2+10a-25}{(a+5)(a-5)(a+2)(a-3)} \\ = \frac{3a-15}{(a+5)(a-5)(a+2)(a-3)} = \frac{3(a-5)}{(a+5)(a-5)(a+2)(a-3)} = \underline{\underline{\frac{3}{(a+5)(a+2)(a-3)}}}$$

90. Die Ideen für die richtigen Nennerfaktorierungen der ersten beiden Brüche ergeben sich aus den Zweiklammeransätzen der hinteren beiden Nenner:

$$x^2 + x - 2 = (x+2)(x-1) \quad \text{und} \quad x^2 - 4x + 3 = (x-1)(x-3)$$

$$\Rightarrow x^4 + 2x^3 - 13x^2 - 14x + 24 = (x-1)(x^3 + 3x^2 - 10x - 24) = (x-1)(x+2)(x^2 + x - 12) \\ = (x-1)(x+2)(x+4)(x-3)$$

$$\Rightarrow x^3 - 2x^2 - 5x + 6 = (x-1)(x^2 - x - 6) = (x-1)(x+2)(x-3)$$

Nun lässt sich die Aufgabe direkt lösen:

$$\frac{6-x}{x^4+2x^3-13x^2-14x+24} + \frac{1}{x^3-2x^2-5x+6} + \frac{1}{x^2+x-2} - \frac{1}{x^2-4x+3} \\ = \frac{6-x}{(x-1)(x+2)(x+4)(x-3)} + \frac{1}{(x-1)(x+2)(x-3)} + \frac{1}{(x+2)(x-1)} - \frac{1}{(x-1)(x-3)} \\ = \frac{6-x+(x+4)+(x+4)(x-3)-(x+2)(x+4)}{(x-1)(x+2)(x+4)(x-3)} = \frac{6-x+x+4+x^2+x-12-x^2-6x-8}{(x-1)(x+2)(x+4)(x-3)} \\ = \frac{-10-5x}{(x-1)(x+2)(x+4)(x-3)} = \frac{-5(x+2)}{(x-1)(x+2)(x+4)(x-3)} = \underline{\underline{-\frac{5}{(x-1)(x+4)(x-3)}}}$$

Multiplikation

91. (a) $3 \cdot \frac{4}{5} = \frac{12}{5}$ (b) $a \cdot \frac{b}{c} = \frac{ab}{c}$ (c) $a \cdot \frac{-b}{c} = \frac{-ab}{c}$ (d) $a \cdot \frac{b}{-c} = \frac{-ab}{c}$ (e) $(-a) \cdot \frac{-b}{c} = \frac{ab}{c}$
92. (a) $x \cdot \frac{y}{x} = \underline{y}$ (b) $u \cdot \frac{v}{v} = \underline{u}$ (c) $n \cdot \frac{m}{n^2} = \frac{m}{n}$ (d) $r^2 \cdot \frac{1}{rs} = \frac{r}{s}$ (e) $pq \cdot \frac{p}{q} = \underline{p^2}$
93. (a) $6ab \cdot \frac{9a}{4b} = \frac{27a^2}{2}$ (b) $44x^2y^2 \cdot \frac{2x^3}{11y^3} = \frac{8x^5}{y}$ (c) $21m^3n \cdot \frac{-7cd}{12mn^2} = \frac{-49cdm^2}{4n}$
94. (a) $29k^5t \cdot \frac{47h^2}{29k^5t} = \underline{47h^2}$ (b) $\frac{5rs^2}{18uv^3} (-15rsuv) = \frac{-25r^2s^3}{6v^2}$ (c) $(-4pz) \left(-\frac{3q^2z}{10p^2} \right) = \frac{6q^2z^2}{5p}$
95. (a) $(a-b) \frac{2a+b}{a-b} = \underline{2a+b}$ (b) $(3x+3y) \cdot \frac{9c}{x+y} = 3(x+y) \frac{9c}{x+y} = \underline{27c}$
(c) $\frac{5}{q^2-1} (q-1) = \frac{5(q-1)}{(q+1)(q-1)} = \frac{5}{q+1}$
96. (a) $4z \cdot \frac{z+1}{8z^2+12z} = \frac{4z(z+1)}{4z(2z+3)} = \frac{z+1}{2z+3}$ (b) $\frac{d}{d^2-8d+15} (d-5) = \frac{d-5}{(d-3)(d-5)} = \frac{1}{d-3}$
(c) $(2k-7) \frac{k}{7-2k} = \underline{-k}$
97. (a) $(r^2-36s^2) \frac{r+6s}{r-6s} = \frac{(r+6s)^2(r-6s)}{r-6s} = \underline{(r+6s)^2}$ (b) $(2p-4) \cdot \frac{p-4}{p^2-4} = \frac{2(p-2)(p-4)}{(p+2)(p-2)} = \frac{2(p-4)}{p+2}$
(c) $\frac{a+b+c}{ab+ac} \cdot abc = \frac{abc(a+b+c)}{a(b+c)} = \frac{bc(a+b+c)}{b+c}$
98. (a) $\frac{x}{yz} (xz+yz) = \frac{xz(x+y)}{yz} = \frac{x(x+y)}{y}$ (b) $(3g-3f) \frac{4f+4g}{5f-5g} = \frac{3(g-f) \cdot 4(f+g)}{5(f-g)} = \underline{-\frac{12(f+g)}{5}}$
(c) $(m^2-n^2) \cdot \frac{(m-n)^2}{(m+n)^2} = \frac{(m+n)(m-n)^3}{(m+n)^2} = \frac{(m-n)^3}{m+n}$
99. (a) $\frac{4}{5} \cdot \frac{7}{3} = \frac{28}{15}$ (b) $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ (c) $\frac{a}{b} \cdot \frac{b}{a} = \underline{1}$ (d) $\frac{a}{b} \cdot \frac{a}{b} = \frac{a^2}{b^2}$ (e) $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$
100. (a) $\frac{a}{b} \cdot \frac{c}{-d} = \frac{-ac}{bd}$ (b) $\frac{a}{-b} \cdot \frac{-c}{d} = \frac{ac}{bd}$ (c) $\left(-\frac{1}{n}\right)^5 = \underline{-\frac{1}{n^5}}$
(d) $\left(-\frac{a}{-b}\right) \left(-\frac{-c}{-d}\right) \left(-\frac{-e}{-f}\right) = \left(-\frac{a}{b}\right) \left(-\frac{c}{d}\right) \left(-\frac{e}{f}\right) = \underline{-\frac{ace}{bdf}}$
101. (a) $\frac{8a}{3b} \cdot \frac{9bc}{4a} = \underline{6c}$ (b) $\frac{-xy^2}{35z^3} \cdot \frac{7z^2}{x^2y^2} = \underline{-\frac{1}{5xz}}$ (c) $\frac{-18u^2w}{65v^4} \cdot \frac{-26v}{27uw^3} = \frac{4u}{15v^3w^2}$
102. (a) $\frac{7m^2}{12n^3} \cdot \frac{-3n^2}{14m} = \underline{-\frac{m}{8n}}$ (b) $\frac{-a}{b} \cdot \frac{-b}{c} \cdot \frac{-c}{a} = \underline{-1}$ (c) $\frac{17r^4s^3}{54t^5} \cdot \frac{24st^2}{85r^2} = \frac{4r^2s^4}{45t^3}$
103. (a) $\left(\frac{6a}{7b}\right)^2 = \frac{36a^2}{49b^2}$ (b) $\left(\frac{-12}{n^3}\right)^2 = \frac{144}{n^6}$ (c) $\left(-\frac{xyz}{cd}\right)^2 = \frac{x^2y^2z^2}{c^2d^2}$ (d) $\left(\frac{m}{4}\right)^3 = \frac{m^3}{64}$
104. (a) $\left(\frac{-8h^2}{9}\right)^2 = \frac{64h^4}{81}$ (b) $\left(\frac{5uv}{17w}\right)^2 = \frac{25u^2v^2}{289w^2}$ (c) $\left(\frac{19r}{2st}\right)^2 = \frac{361r^2}{4s^2t^2}$ (d) $\left(-\frac{3}{e^2}\right)^4 = \frac{81}{e^8}$
105. (a) $\frac{m-n}{3m} \cdot \frac{5m}{2m-2n} = \underline{\frac{5}{6}}$ (b) $\frac{d-1}{18d} \cdot \frac{12d^2}{1-d} = \underline{-\frac{2d}{3}}$ (c) $\frac{x^2+y^2}{x^2-y^2} \cdot \frac{x-y}{xy} = \frac{x^2+y^2}{(x+y)(x-y)} \cdot \frac{x-y}{xy} = \underline{\frac{x^2+y^2}{xy(x+y)}}$
106. (a) $\frac{t}{4u+4v} \cdot \frac{3u^2-3v^2}{t^2+t} = \frac{t}{4(u+v)} \cdot \frac{3(u+v)(u-v)}{t(t+1)} = \frac{3(u-v)}{4(t+1)}$ (b) $\frac{5a^2}{5b-3} \cdot \frac{9-15b}{10ac} = \frac{5a^2}{5b-3} \cdot \frac{-3(5b-3)}{10ac} = \underline{-\frac{3a}{2c}}$
(c) $\frac{7r^2s}{12(r-s)} \cdot \frac{(2s-2r)^2}{21rs^2} = \frac{7r^2s}{12(r-s)} \cdot \frac{4(r-s)^2}{21rs^2} = \underline{\frac{r(r-s)}{9s}}$
107. (a) $\frac{p^2-q^2}{p^2+q^2} \cdot \frac{p+q}{p-q} = \frac{(p+q)(p-q)}{p^2+q^2} \cdot \frac{p+q}{p-q} = \frac{(p+q)^2}{p^2+q^2}$
(b) $\frac{x^2-6xy+9y^2-z^2}{5m-5n} \cdot \frac{m^4-n^4}{x-3y+z} = \frac{(x-3y+z)(x-3y-z)}{5(m-n)} \cdot \frac{(m^2+n^2)(m+n)(m-n)}{x-3y+z} = \underline{\frac{(x-3y-z)(m^2+n^2)(m+n)}{5}}$
108. (a) $\frac{v^2+4v+4}{3t-3} \cdot \frac{9-9t}{v^2+5v+6} = \frac{(v+2)^2}{3(t-1)} \cdot \frac{9(1-t)}{(v+2)(v+3)} = \underline{-\frac{3(v+2)}{v+3}}$
(b) $\frac{a^3-3a^2+3a-1}{225a^2b^2-150abc+25c^2} \cdot \frac{45abc-15c^2}{ab-b} = \frac{(a-1)^3}{25(3ab-c)^2} \cdot \frac{15c(3ab-c)}{b(a-1)} = \underline{\frac{3c(a-1)^2}{5b(3ab-c)}}$

109. (a) $xy \left(\frac{x}{y} + \frac{y}{x} \right) = xy \cdot \frac{x^2+y^2}{xy} = \underline{\underline{x^2 + y^2}}$ (b) $(n-z) \left(\frac{n}{n-z} - \frac{z}{n^2-z^2} \right) = (n-z) \cdot \frac{n(n+z)-z}{(n+z)(n-z)} = \underline{\underline{\frac{n^2+nz-z}{n+z}}}$
(c) $\left(-\frac{r^2}{s^2} \right) \left(\frac{s}{r} - \frac{s^2}{r^2} + \frac{s^3}{r^3} \right) = -\frac{r}{s} + 1 - \frac{s}{r} = \underline{\underline{\frac{-r^2+rs-s^2}{rs}}} = \underline{\underline{\frac{-r^2-rs+s^2}{rs}}}$
110. (a) $\left(c - \frac{d}{c} \right) \left(c + \frac{d}{c} \right) = \frac{c^2-d}{c} \cdot \frac{c^2+d}{c} = \underline{\underline{\frac{(c^2-d)(c^2+d)}{c^2}}}$
(b) $\frac{u^2-v^2}{u^2+v^2} \left(\frac{u}{u+v} + \frac{v}{u-v} \right) = \frac{(u+v)(u-v)}{u^2+v^2} \cdot \frac{u(u-v)+v(u+v)}{(u+v)(u-v)} = \frac{u^2-uv+uv+v^2}{u^2+v^2} = \underline{\underline{1}}$
(c) $\left(\frac{ab}{a-b} + a \right) \left(\frac{ab}{a+b} - b \right) \frac{b-a}{ab^2} = \frac{ab+a^2-ab}{a-b} \cdot \frac{ab-ab-b^2}{a+b} \cdot \frac{b-a}{ab^2} = \frac{a^2b^2(b-a)}{ab^2(a+b)(a-b)} = \underline{\underline{-\frac{a}{a+b}}}$
111. (a) $\left(\frac{n}{2} - \frac{1}{n} \right)^2 = \left(\frac{n^2-2}{2n} \right)^2 = \underline{\underline{\frac{(n^2-2)^2}{4n^2}}}$ (b) $\left(\frac{z^2}{x-z} + z \right)^2 = \left(\frac{z^2+xz-z^2}{x-z} \right)^2 = \underline{\underline{\frac{x^2z^2}{(x-z)^2}}}$
(c) $\left(\frac{p}{q} - 1 \right)^2 - \left(\frac{p}{q} + 1 \right)^2 = \frac{(p-q)^2}{q^2} - \frac{(p+q)^2}{q^2} = \frac{p^2-2pq+q^2-p^2-2pq-q^2}{q^2} = -\frac{4pq}{q^2} = \underline{\underline{-\frac{4p}{q}}}$
112. (a) $\left(\frac{a}{2b} - \frac{c}{3d} \right)^2 = \left(\frac{3ad-2bc}{6bd} \right)^2 = \underline{\underline{\frac{(3ad-2bc)^2}{36b^2d^2}}}$
(b) $\left(\frac{1}{r-s} - \frac{1}{r+s} \right)^2 = \left(\frac{r+s-r+s}{(r+s)(r-s)} \right)^2 = \left(\frac{2s}{(r+s)(r-s)} \right)^2 = \underline{\underline{\frac{4s^2}{(r+s)^2(r-s)^2}}}$
(c) $\left(u - \frac{v}{u} \right)^2 - \left(u + \frac{v}{u} \right)^2 = \frac{(u^2-v)^2 - (u^2+v)^2}{u^2} = \frac{u^4-2u^2v+v^2-u^4-2u^2v-v^2}{u^2} = \frac{-4u^2v}{u^2} = \underline{\underline{-4v}}$
113. $\left(\frac{x}{3} - \frac{y}{2} \right) \left(\frac{x}{2} + y \right) - \left(\frac{x}{3} + y \right) \left(\frac{x}{2} - y \right) = \frac{2x-3y}{6} \cdot \frac{x+2y}{2} - \frac{x+3y}{3} \cdot \frac{x-2y}{2}$
 $= \frac{(2x-3y)(x+2y)}{12} - \frac{2(x+3y)(x-2y)}{12} = \frac{2x^2+xy-6y^2-2x^2-2xy+12y^2}{12} = \frac{-xy+6y^2}{12} = \underline{\underline{\frac{y(-x+6y)}{12}}}$
114. $\frac{3a}{3a-2b} \cdot \frac{3a}{2b} - \left(\frac{3a}{3a-2b} + \frac{3a}{2b} \right) = \frac{9a^2}{2b(3a-2b)} - \frac{6ab+9a^2-6ab}{2b(3a-2b)} = \frac{9a^2-9a^2}{2b(3a-2b)} = \underline{\underline{0}}$
115. (a) $\left(1 + \frac{r}{s} \right)^3 - \left(1 - \frac{r}{s} \right)^3 = 1 + \frac{3r}{s} + \frac{3r^2}{s^2} + \frac{r^3}{s^3} - \left(1 - \frac{3r}{s} + \frac{3r^2}{s^2} - \frac{r^3}{s^3} \right) = \frac{6r}{s} + \frac{2r^3}{s^3} = \frac{6rs^2+2r^3}{s^3} = \underline{\underline{\frac{2r(r^2+3s^2)}{s^3}}}$
(b) $\left(\frac{n^3-2n-1}{n^2-1} - n \right) \left(n - \frac{2n^2}{n+1} \right) = \frac{n^3-2n-1-n^3+n}{n^2-1} \cdot \frac{n^2+n-2n^2}{n+1} = \frac{-n-1}{(n+1)(n-1)} \cdot \frac{-n^2+n}{n+1}$
 $= \frac{-(n+1)}{(n+1)(n-1)} \cdot \frac{-n(n-1)}{n+1} = \underline{\underline{\frac{n}{n+1}}}$
116. (a) $\left(\frac{c}{3} - 1 \right)^3 - \left(\frac{c}{3} + 1 \right)^3 = \frac{c^3}{27} - \frac{c^2}{3} + c - 1 - \left(\frac{c^3}{27} + \frac{c^2}{3} + c + 1 \right) = -\frac{2c^3}{3} - 2 = \underline{\underline{\frac{-2c^3-6}{3}}} = \underline{\underline{-\frac{2c^3+3}{3}}}$
(b) $\left(\frac{8x^2+4x+1}{4x^2-2x} - \frac{2x}{2x-1} \right) \frac{6x-3}{4x^2+2x} = \frac{8x^2+4x+1-4x^2}{2x(2x-1)} \cdot \frac{3(2x-1)}{2x(2x+1)} = \frac{(4x^2+4x+1) \cdot 3(2x-1)}{4x^2(2x+1)(2x-1)}$
 $= \frac{3(2x+1)^2(2x-1)}{4x^2(2x+1)(2x-1)} = \underline{\underline{\frac{3(2x+1)}{4x^2}}}$