Prüfung 1 Exponentialgleichungen und Logarithmen – Lösungen

1. Für die Lösungen erhalten wir: (total 4 P)

   (a)$_1$ $\quad 5^{6x} = 125 \quad \Leftrightarrow \quad 5^{6x} = 5^3 \quad \Leftrightarrow \quad 6x = 3 \quad \Leftrightarrow \quad \underline{\underline{x = \frac{1}{2}}}$

   (b)$_{1.5}$ $\quad 3^{4x-1} = 5^x \quad \Leftrightarrow \quad 4x - 1 = \log_3(5^x) \quad \Leftrightarrow \quad 4x - 1 = x\log_3(5) \quad \Leftrightarrow \quad \underline{\underline{x = \frac{1}{4 - \log_3(5)}}}$

   (c)$_{1.5}$ $\quad (5x+1)^{\frac{3}{2}} = 64 \quad \Leftrightarrow \quad 5x + 1 = 64^{\frac{2}{3}} = 16 \quad \Leftrightarrow \quad 5x = 15 \quad \Leftrightarrow \quad \underline{\underline{x = 3}}$

2. Wir berechnen: (total 8 P)

   (a)$_1$ $\quad \log_3(81) = \underline{\underline{4}}$ (b)$_1$ $\quad \log_{16}(32) = \underline{\underline{\frac{5}{4}}}$

   (c)$_{1.5}$ $\quad \log\left(\frac{10}{\sqrt[5]{10}}\right) = \log\left(10^{1-\frac{1}{5}}\right) = \underline{\underline{\frac{4}{5}}}$

   (d)$_{1.5}$ $\quad 10^{-\frac{3}{2}\log(25)} = 10^{\log\left(25^{-\frac{3}{2}}\right)} = 25^{-\frac{3}{2}} = \underline{\underline{\frac{1}{125}}}$

   (e)$_{1.5}$ $\quad 9^{\log_3(\sqrt{5}\,)} = \left(3^2\right)^{\log_3(\sqrt{5}\,)} = 3^{2\log_3(\sqrt{5}\,)} = 3^{\log_3(5)} = \underline{\underline{5}}$

   (f)$_{1.5}$ $\quad \ln\left(\ln\left(e^{\sqrt{e^5}}\right)\right) = \ln\left(\sqrt{e^5}\,\right) = \ln\left(e^{\frac{5}{2}}\right) = \underline{\underline{\frac{5}{2}}}$

3. Wir vereinfachen: (total 5 P)

   (a)$_{1.5}$ $\quad \lg(5) - \lg(32) + \lg(8) - \lg(125) = \lg\left(\frac{5 \cdot 8}{32 \cdot 125}\right) = \lg\left(\frac{1}{4 \cdot 25}\right) = \lg\left(\frac{1}{100}\right) = \underline{\underline{-2}}$

   (b)$_2$ $\quad 3\log_6(25) - \log_6(625) - 4\log_6\left(\sqrt{5}\,\right) = \log_6\left(\frac{25^3}{625 \cdot \left(\sqrt{5}\,\right)^4}\right) = \log_6\left(\frac{5^6}{5^4 \cdot 5^2}\right) = \log_6(1) = \underline{\underline{0}}$

   (c)$_{1.5}$ $\quad \ln(3) + \ln(3) - 1 = 2\ln(3) - \ln(e) = \ln\left(\frac{3^2}{e}\right) = \underline{\underline{\ln\left(\frac{9}{e}\right)}}$

4. Für die Zerlegung ergibt sich: (2 P)

$$2\log\left(\frac{10c^3}{1000d}\right) = 2\log\left(\frac{c^3}{100d}\right) = 2\left(\log\left(c^3\right) - \log(100) - \log(d)\right) = \underline{\underline{6\log(c) - 4 - 2\log(d)}}$$

5. Wir lösen: (total 9 P)

   (a)$_2$ $\quad 3^{x+1} + 3^{x-1} = 90 \quad \Leftrightarrow \quad 3^x\left(3 + \frac{1}{3}\right) = 3^x \cdot \frac{10}{3} = 90 \quad \Leftrightarrow \quad 3^x = \frac{270}{10} = 27 \quad \Leftrightarrow \quad \underline{\underline{x = 3}}$

   (b)$_2$ $\quad 2^x + 4^x = 12 \quad \Leftrightarrow \quad 2^x + \left(2^2\right)^x = 12 \quad \Leftrightarrow \quad \left(2^x\right)^2 + 2^x - 12 = 0 \quad \Leftrightarrow \quad \left(2^x + 4\right)\left(2^x - 3\right) = 0$

   $\Rightarrow$ Fall 1: $2^x = -4$ $\rightarrow$ geht nicht! Fall 2: $2^x = 3 \quad \Leftrightarrow \quad \underline{\underline{x = \log_2(3)}}$

   (c)$_{2.5}$ $\quad \log\left(x^2 + 99\right) - \log(x) = 2 \quad \Leftrightarrow \quad \log\left(\frac{x^2+99}{x}\right) = 2 \quad \Leftrightarrow \quad \frac{x^2+99}{x} = 10^2 = 100$

   $\Leftrightarrow \quad x^2 + 99 = 100x \quad \Leftrightarrow \quad x^2 - 100x + 99 = 0 \quad \Leftrightarrow \quad (x-1)(x-99) = 0 \quad \Leftrightarrow \quad \underline{\underline{\mathbb{L} = \{1, 99\}}}$

   (d)$_{2.5}$ $\quad \log_2(x) = 1 + \log_4(x) \quad \Leftrightarrow \quad \log_2(x) = \log_4(4) + \log_4(x) = \log_4(4x) = \frac{\log_2(4x)}{\log_2(4)} = \frac{\log_2(4x)}{2}$

   $\Leftrightarrow \quad 2\log_2(x) = \log_2(4x) \quad \Leftrightarrow \quad \log_2\left(x^2\right) = \log_2(4x) \quad \Rightarrow \quad x^2 = 4x \quad \Leftrightarrow \quad x(x-4) = 0$

   $\Rightarrow$ Fall 1: $x = 0$ $\rightarrow$ nicht erlaubt! Fall 2: $\underline{\underline{x = 4}}$

6. Wir benutzen die Basiswechselformel und formen ganz direkt um: (1 P)

$$\log_{\sqrt[n]{a}}(b) = \frac{\log_a(b)}{\log_a\left(\sqrt[n]{a}\,\right)} = \frac{\log_a(b)}{\frac{1}{n}} = n\log_a(b) \qquad \text{q.e.d.}$$