Übungen zur Differentialrechnung – Lösungen Serie II

1. Es ergeben sich die folgenden Ableitungen:

$$a(x) = 2x + 5 \quad \Rightarrow \quad \underline{\underline{a'(x) = 2}} \qquad\qquad b(x) = 7x^2 - 5x + 29 \quad \Rightarrow \quad \underline{\underline{b'(x) = 14x - 5}}$$

$$c(x) = 13x^3 - 8x^2 + 15x - 14 \quad \Rightarrow \quad \underline{\underline{c'(x) = 39x^2 - 16x + 15}}$$

$$d(x) = 12x^5 - 9x^3 \quad \Rightarrow \quad \underline{\underline{d'(x) = 60x^4 - 27x^2}} \qquad\qquad e(x) = 155c \quad \Rightarrow \quad \underline{\underline{e'(x) = 0}}$$

$$f(x) = \frac{1}{6}\,x^3 - \frac{1}{4}\,x^2 + \frac{1}{3}\,x - \frac{1}{100} \quad \Rightarrow \quad \underline{\underline{f'(x) = \frac{1}{2}\,x^2 - \frac{1}{2}\,x + \frac{1}{3}}}$$

$$g(x) = 6x^6 - \frac{1}{5}\,x^5 \quad \Rightarrow \quad \underline{\underline{g'(x) = 36x^5 - x^4}}$$

$$h(x) = -\frac{1}{23}\,x^8 - \frac{3}{10}\,x^5 \quad \Rightarrow \quad \underline{\underline{h'(x) = -\frac{8}{23}\,x^7 - \frac{3}{2}\,x^4}}$$

$$i(x) = \frac{4}{81}\,x^{27} - \frac{5}{9}\,x^{12} + \frac{12}{25}\,x^5 + \pi \quad \Rightarrow \quad \underline{\underline{i'(x) = \frac{4}{3}\,x^{26} - \frac{20}{3}\,x^{11} + \frac{12}{5}\,x^4}}$$

2. Hier lauten die Ableitungsfunktionen wie folgt:

$$a(x) = ax^3 + bx^2 + cx + d \quad \Rightarrow \quad \underline{\underline{a'(x) = 3ax^2 + 2bx + c}}$$

$$b(x) = mx^3 - 7qx + 8z \quad \Rightarrow \quad \underline{\underline{b'(x) = 3mx^2 - 7q}}$$

$$c(x) = 2(a+d)x^6 - 7(v-3)x^5 \quad \Rightarrow \quad \underline{\underline{c'(x) = 12(a+d)x^5 - 35(v-3)x^4}}$$

$$d(x) = \frac{b+c}{14}\,x^7 - \frac{b-c}{12}\,x^4 \quad \Rightarrow \quad \underline{\underline{d'(x) = \frac{b+c}{2}\,x^6 - \frac{b-c}{3}\,x^3}}$$

$$e(x) = \frac{1}{n}\,x^n \quad \Rightarrow \quad \underline{\underline{e'(x) = x^{n-1}}}$$

$$f(x) = \frac{m}{np}\,x^p + \frac{m}{n}\,x^m \quad \Rightarrow \quad \underline{\underline{f'(x) = \frac{m}{n}\,x^{p-1} + \frac{m^2}{n}x^{m-1}}}$$

3. Wir wenden das Rezept an und erhalten für die Ableitungen:

$$a(x) = \frac{1}{x^2} = x^{-2} \quad \Rightarrow \quad a'(x) = -2 \cdot x^{-3} = \underline{\underline{-\frac{2}{x^3}}}$$

$$b(x) = \frac{3}{x^3} = 3 \cdot x^{-3} \quad \Rightarrow \quad b'(x) = -9 \cdot x^{-4} = \underline{\underline{-\frac{9}{x^4}}}$$

$$c(x) = -\frac{1}{x^4} = -x^{-4} \quad \Rightarrow \quad c'(x) = 4 \cdot x^{-5} = \underline{\underline{\frac{4}{x^5}}}$$

$$d(x) = \frac{2}{5x^5} = \frac{2}{5}\,x^{-5} \quad \Rightarrow \quad d'(x) = -2 \cdot x^{-6} = \underline{\underline{-\frac{2}{x^6}}}$$

$$e(x) = \sqrt{2} \quad \Rightarrow \quad e'(x) = \underline{\underline{0}}$$

$$f(x) = 12\sqrt[3]{x} = 12x^{\frac{1}{3}} \quad \Rightarrow \quad f'(x) = 4x^{-\frac{2}{3}} = \underline{\underline{\frac{4}{\sqrt[3]{x^2}}}}$$

$$g(x) = 12\sqrt[4]{x} = 12x^{\frac{1}{4}} \quad \Rightarrow \quad g'(x) = 3x^{-\frac{3}{4}} = \underline{\underline{\frac{3}{\sqrt[4]{x^3}}}}$$

$$h(x) = \frac{2}{\sqrt{x}} = 2x^{-\frac{1}{2}} \quad \Rightarrow \quad h'(x) = -x^{-\frac{3}{2}} = -\frac{1}{\sqrt{x^3}} = \underline{\underline{-\frac{1}{x\sqrt{x}}}}$$

$$i(x) = \sqrt[3]{\frac{1}{x}} = x^{-\frac{1}{3}} \quad \Rightarrow \quad i'(x) = -\frac{1}{3}\,x^{-\frac{4}{3}} = -\frac{1}{3\sqrt[3]{x^4}} = \underline{\underline{-\frac{1}{3x\sqrt[3]{x}}}}$$

$$j(x) = x^2\sqrt{x} = x^{\frac{5}{2}} \quad \Rightarrow \quad j'(x) = \frac{5}{2}\,x^{\frac{3}{2}} = \frac{5}{2}\,\sqrt{x^3} = \underline{\underline{\frac{5}{2}\,x\sqrt{x}}}$$

$$k(x) = \frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}} \quad \Rightarrow \quad k'(x) = -\frac{2}{3}\,x^{-\frac{5}{3}} = -\frac{2}{3\sqrt[3]{x^5}} = \underline{\underline{-\frac{2}{3x\sqrt[3]{x^2}}}}$$

$$l(x) = \frac{4}{5x\sqrt[4]{x}} = \frac{4}{5}\,x^{-\frac{5}{4}} \quad \Rightarrow \quad l'(x) = -x^{-\frac{9}{4}} = -\frac{1}{\sqrt[4]{x^9}} = \underline{\underline{-\frac{1}{x^2\sqrt[4]{x}}}}$$

$$m(x) = \frac{155\sqrt{x^{155}}}{x^{76}} = 155x^{\frac{155}{2}-76} = 155x^{\frac{3}{2}} \quad \Rightarrow \quad m'(x) = \frac{3}{2}\cdot 155x^{\frac{1}{2}} = \underline{\underline{\frac{465}{2}\,\sqrt{x}}}$$

$$n(x) = 3\sqrt{x} + x\sqrt{x} = 3x^{\frac{1}{2}} + x^{\frac{3}{2}} \quad \Rightarrow \quad n'(x) = \frac{3}{2}\,x^{-\frac{1}{2}} + \frac{3}{2}\,x^{\frac{1}{2}} = \underline{\underline{\frac{3}{2\sqrt{x}} + \frac{3}{2}\,\sqrt{x}}}$$

4. Es ergeben sich die folgenden Rechenschritte resp. Tangentengleichungen:

$$f(x) = 5x^2 \quad \text{mit} \quad f'(x) = 10x \quad \Rightarrow \quad f(3) = 45 \quad \text{und} \quad f'(3) = 30$$
$$\Rightarrow \quad t_f(x) = 30(x-3) + 45 = 30x - 90 + 45 = \underline{\underline{30x - 45}}$$

$$g(x) = -x^3 + 4x - 5 \quad \text{mit} \quad g'(x) = -3x^2 + 4 \quad \Rightarrow \quad g(1) = -2 \qquad \text{und} \quad g'(1) = 1$$
$$\Rightarrow \quad t_g(x) = 1(x-1) - 2 = x - 1 - 2 = \underline{\underline{x - 3}}$$

$$h(x) = \frac{1}{x^2} \quad \text{mit} \quad h'(x) = -\frac{2}{x^3} \quad \Rightarrow \quad h(-2) = \frac{1}{4} \quad \text{und} \quad h'(-2) = \frac{1}{4}$$
$$\Rightarrow \quad t_h(x) = \frac{1}{4}\,(x+2) + \frac{1}{4} = \frac{1}{4}\,x + \frac{1}{2} + \frac{1}{4} = \underline{\underline{\frac{1}{4}\,x + \frac{3}{4}}}$$

$$i(x) = 2\sqrt{x} \quad \text{mit} \quad i'(x) = \frac{1}{\sqrt{x}} \quad \Rightarrow \quad i(4) = 4 \quad \text{und} \quad i'(4) = \frac{1}{2}$$
$$\Rightarrow \quad t_i(x) = \frac{1}{2}\,(x-4) + 4 = \frac{1}{2}\,x - 2 + 4 = \underline{\underline{\frac{1}{2}\,x + 2}}$$

$$j(x) = \frac{2}{x\sqrt[3]{x}} = 2x^{-\frac{4}{3}} \quad \text{mit} \quad j'(x) = -\frac{8}{3}x^{-\frac{7}{3}} = -\frac{8}{3x^2\sqrt[3]{x}} \quad \Rightarrow \quad j(-1) = 2 \quad \text{und} \quad j'(-1) = \frac{8}{3}$$
$$\Rightarrow \quad t_j(x) = \frac{8}{3}\,(x+1) + 2 = \frac{8}{3}\,x + \frac{8}{3} + 2 = \underline{\underline{\frac{8}{3}\,x + \frac{14}{3}}}$$

$$k(x) = 2x - \frac{3}{x} \quad \text{mit} \quad k'(x) = 2 + \frac{3}{x^2} \quad \Rightarrow \quad k(2) = \frac{5}{2} \quad \text{und} \quad k'(2) = \frac{11}{4}$$
$$\Rightarrow \quad t_k(x) = \frac{11}{4}\,(x-2) + \frac{5}{2} = \frac{11}{4}\,x - \frac{11}{2} + \frac{5}{2} = \underline{\underline{\frac{11}{4}\,x - 3}}$$